

Central Tendency Exercises

<u>Pages</u>	<u>Suggested Reading</u>
69 – 73	Sections 2.7 and 2.8

<u>Pages</u>	<u>Problems</u>
87 – 100	(Section 2.13) 2 , 8(a,c) , 17(a,g,i,j,k), 19, 25

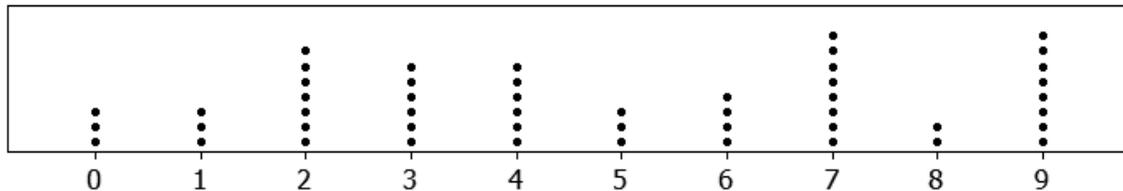
<u>Addenda Videos</u>
Why grouping data into a histogram doesn't lose THAT much precision when averaging.

For E1 and E2, please look again at the dataset from question 8 again.

E1. If you double each data point, what happens to the mean? Median?

E2. If you decrease each data point by 5, what happens to the mean? Median?

E3. What are the mean, median and mode for the following data? Try to do it strictly using the dotplot first (that is, without using formulas or listing data in your TI).



*“More than almost 14 months ago, I was calling for a trickle-up approach for homeowners in this country. At that point, the math is so simple. Average price of a house was \$206,000. A million [underwater] houses...that would be [about] \$200 billion. If we had written checks from the treasury and paid off those mortgages, we would **still** have \$8.3 trillion to apply to the other issues in this country. I mean, what is going **on** in that Congress?”* (source: Lou Dobbs Show, 12.10.08)

E4. Assuming that Lou’s data are right (that is, the average home price was \$206,000, and there were a million of them that were underwater), is his number of \$200 billion dollars correct? Why or why not?

Suppose a student earns the following transcript for his Fall 2009 term:

<u>Class</u>	<u>Credits</u>	<u>Grade</u>
Statistics	4	A
Economics	3	B+
US History	3	C
Weightlifting	1	A

E5. Calculate his Fall 2009 Grade Point Average (GPA). Assume that a grade of “A” holds a numerical value of 4, “B+” holds a value of 3.33, and “C” holds a value of 2. Round to the hundredths’ place.

E6. Refer back to the previous question. Suppose that this student has been at COCC for a number of terms. His cumulative GPA (*not including* Fall 2009) for all of his time at COCC is 3.6. In this cumulative time, he took 58 credits (remember, not including Fall 2009). Calculate his cumulative GPA of his entire COCC career, *including* Fall 2009. Again, round to the nearest hundredths' place.

E7. OK...now it's time to plan for Winter term, 2010. The student would like to have a GPA of 3.6 at the end of this term. He'll be taking 12 credits. Is it possible? Why or why not?

Answers.

E1. They are both doubled. Here are two videos to help you: [one using Excel](#), and [one using algebra](#).

E2. They are both decreased by 5. Here's a [video to help you understand it!](#)

E3. I get a mean of about 4.5, a median, also, of 4.5, and modes of 7 and 9, all by approximating from the dotplot. Did anyone crunch the numbers? How close did I get?

E4. Yes. As to *why*, look again at how an average is calculated. In that formula, there are three components; he's given you all three.

E5. It's a weighted mean, so you can set it up like we did in class:

<u>Class</u>	<u>Grade (x)</u>	<u>Credits (weights...or frequencies, if you like)</u>	<u>xf</u>
Statistics	A = 4	4	4*4 = 16
Economics	B+ = 3.33	3	3.33*3 = 9.99
US History	C = 2	3	2*3 = 6
Weightlifting	A = 4	1	4*1 = 4

So, the GPA would be $\frac{\sum xf}{\sum f}$, or about 3.27.

E6. Again, it's a weighted mean. Let's take a look:

<u>Time Period</u>	<u>GPA (x)</u>	<u>Credits (weights)</u>	<u>xf</u>
Pre Fall 2011	3.6	58	208.8
Fall 2011	3.27	11	35.97 (it's actually 36, but whatever)

So, the cumulative GPA, $\frac{\sum xf}{\sum f}$, would be 3.55.

E7. Approach it like the last one:

<u>Time Period</u>	<u>GPA (x)</u>	<u>Credits (weights)</u>	<u>xf</u>
Pre Winter 2012	3.55	69	244.95
Winter 2012	?	12	12*?

He would like $\frac{\sum xf}{\sum f}$ to equal 3.6:

$\frac{\sum xf}{\sum f} = \frac{244.95 + 12?}{81} = 3.6$. Solving that for the "?", we see that he'd have to have a W12 GPA of about 3.89. So it's possible (but you'd have to ask him how *likely* it would be!).

.....

Central Tendency Quizzes

Quiz 1.

Remember that any question marked with a (w) requires you to show/explain how you arrived at your answer. If you used technology, explain exactly what you input, and which functions you used.

A neighbor of mine became alarmed at the number of cars running a certain stop sign. For 30 days, he counted the number of cars each day that ran this stop sign. His daily results are summarized below:

0 2 0 5 5 1 5 3 15 1 2 1 11 36 2
21 5 8 14 0 12 0 2 3 3 0 7 5 0 21

1. (1 point each) Find the mean, median and mode of this data.

Number of Cars	X	Frequency
0 – 4		
5 – 9		
10 – 14		
15 – 19		
20 – 24		
25 – 29		
30 – 34		
35 – 39		

- (1 point) Fill in the frequency distribution at left with the correct frequencies, using the same data above.
- (2 points) (w) Find the weighted mean of the distribution at left. I left you a column for the class mark (x). You're welcome. 😊
- (1 point) By what percent larger is this mean than the raw data's mean? ¹

OK...let's group the data differently. I redid the data distribution at right, using larger classes.

- (2 points) (w) Find the weighted mean of the distribution at right.
- (1 point) By what percent larger is *this* mean than the raw data's mean?

Number of Cars	x	Frequency
0 – 9		23
10 – 19		4
20 – 29		2
30 – 39		1

If you regroup the data as follows:

Number of Cars	x	Frequency (f)
0 – 19	9.5	27
20 – 39	29.5	3

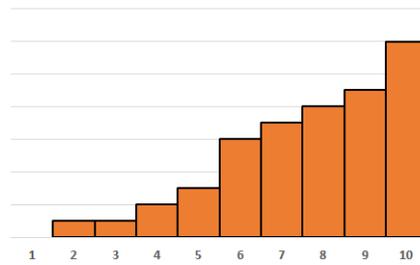
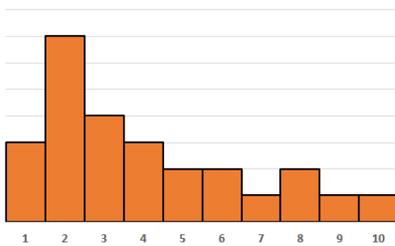
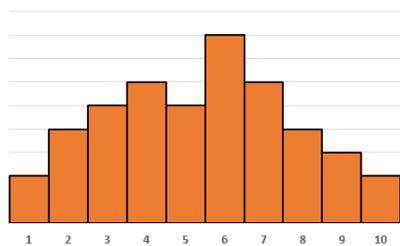
You'll see that the average of *that* distribution is 11.5, which is a whopping **82% bigger** than the true mean!

And therein lies the challenge – grouping data is a convenient way to present it, and a great way to analyze it...but you must take care to ensure that you don't overly generalize the data's behavior at the sake of unintentional bias. You'll get lots of practice doing this in your careers!

¹ A nice way to do this is to divide this mean by the mean you got in number 1. The decimal part of that result indicates the percent overage.

Quiz 2.

In class, you may remember (or maybe not!) that I referred to the average as a “balance point”. If you’re looking at the graph of data, and want to get a good idea of where the average is, imagine the data as a plate of cookies...where would you have to place a finger to balance that tray? For example, for a few different data sets:



What this quiz will do is try and tease out **why** the balance point is the average. Start by thinking back to when we had lollipops in class...best day! That day, I asked, “What’s the average number of lollipops that you have as a group – **without** using the formula ‘add all the data values up and divide by how many data values you have’.”

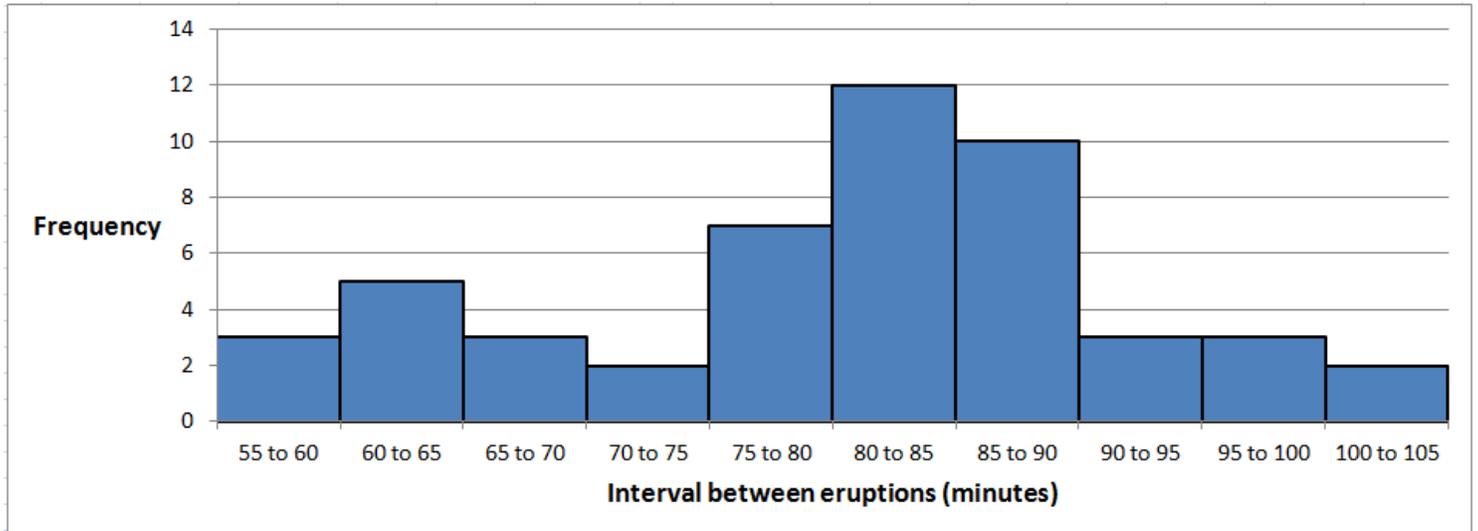
And you DID! By “sharing” your lollipops, you arranged the total number of lollipops (whiteout really knowing what the total number **was**) until everyone had the same number. That “same number” **is** the average.

Now – why is the “sharing number” the same as the balance point? Here we go!

Quiz 3.

Remember that any question marked with a (w) requires you to show/explain how you arrived at your answer. If you used technology, explain exactly what you input, and which functions you used.

The histogram below represents randomly selected times between eruptions of [Old Faithful geyser](#) in Yellowstone National Park. Use this histogram to answer the following questions:



1. (1 point) How many observations are represented in the histogram?
2. (2 points) What is the average length of time between eruptions?
3. (2 points) What is the median length of time between eruptions?
4. (1 point) Yellowstone rangers, in 2001, told me “Old Faithful erupts every 80 to 90 minutes.” What percentage of the intervals are between 80 and 90 minutes in length (assuming that the data above represents a good random sample of eruption intervals)?
5. (1 point) What percentage of the intervals are less than 80 minutes in length?
6. (1 point) What percentage of the intervals are more than 90 minutes in length?
7. (2 points) Based on your answers, do you think the geyser’s name (“Old Faithful”) is appropriate? Why or why not?

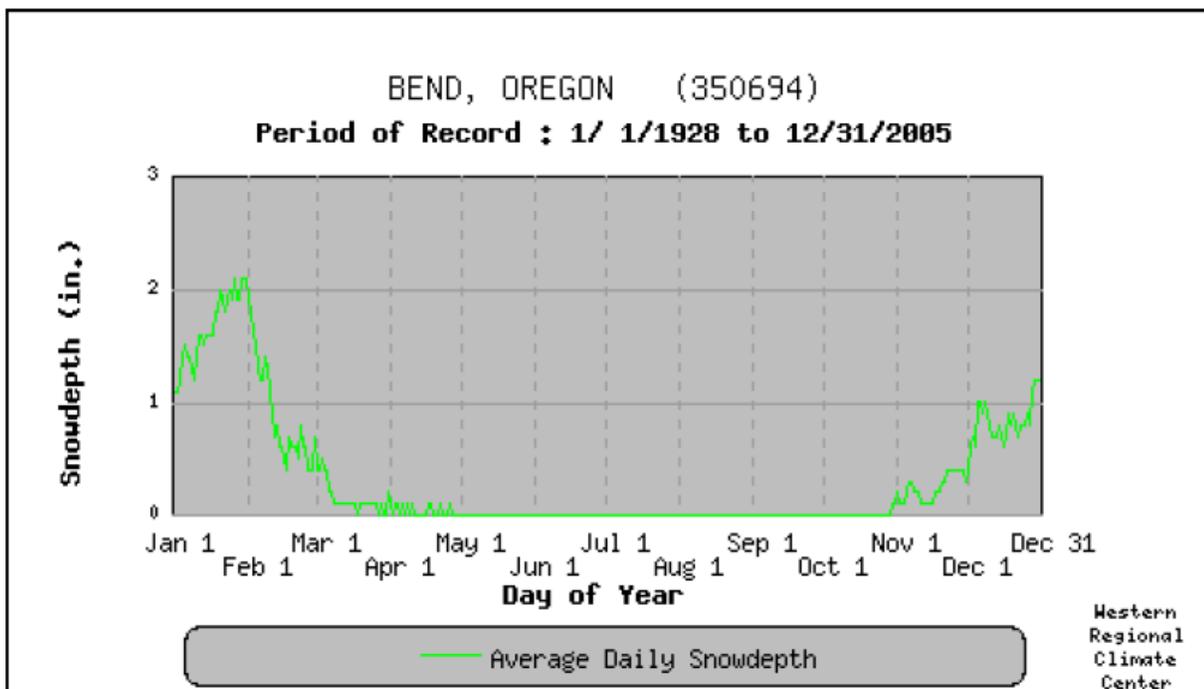


Quiz 4.

On February 8, 2014, my yard got 25" of snow before the day was over. Hell YEAH! I'd like to see how that changes the average historical snow depth in Bend on February 8. I checked in with my good friends at the Western Regional Climate Center (<http://www.wrcc.dri.edu/cgi-bin/cliMAIN.pl?or0699>).

(5 points) (w) From 1991 to 2013, the mean snow depth on February 8th was about 0.2". Find the new average snow depth including the 25" on 2.8.14 (assuming that Bend got, uniformly, 25" – a stretch, I know). Remember that this is a weighted average! There are 23 years between, and including, 1991 and 2013). If you need help on this and the next one, the GPA examples from the central tendency homework will assist you nicely! Let's go to the nearest tenth of an inch.

I have a much more all – inclusive graphic that stretches from 1928 to 2005 (you saw it in class):



Based on this graphic, it looks like the average annual snow depth on February 8th was a little more between these 78 years (around 0.75") than it was from 1991 to 2013.

(5 points) (w) Find the new average snow depth on February 8th from 1928 to 2014 (you can ignore the years from 2006 to 2013) including the 25" on 2.8.14. Same rounding.

.....

Quiz 5.

1. **(5 points)** You're hiking in the backcountry of Oregon, and encounter a wide river. You're nervous, as you can't swim very well. You'd like to know if you can safely cross this river by foot, so you ask another hiker you see. He tells you the depth of the river, on average, is 2 feet. You think to yourself, "Well, since I'm way more than 2 feet tall, I should be able to cross this river safely." Explain clearly why you might be wrong (assume that the hiker told the truth; that is, the river's average depth *is* indeed 2 feet). You may draw a picture to help you explain, if you like. Also, don't worry about the speed of the water in the river for this problem – although definitely **DO** if you're ever thinking of wading across a river **anywhere**.
2. **(5 points)** In 1970, Paul Anthony Samuelson won the Nobel Memorial Prize in Economic Sciences (in only its second year). When giving an interview, Samuelson made the following statement (paraphrased): "I was a physics major before I was an economics major. When I switched from physics to economics, I raised the average IQ of both physicists and economists." How is this possible? Assume his IQ didn't change when he became an economist.

Quiz 6.

Here's another to show you how powerful an "average" can be...

Each year, most US residents have to "do their taxes" to find out how much money they either a) owe the federal government, or b) will be getting back from said government.

1. **(points)** Do a little Googling and find out what the most recent average tax return is in the US. Please also supply your source.
2. **(points)** Do a little more Googling and see how many tax returns were filed during that same time period.

Now, since you have both the sum total of tax returns submitted, and the average dollar amount per tax return, you can figure out just how much money was redistributed.

3. **(points)** Do that now! Hint: **average** = $\frac{\text{sum of all data}}{\text{number of data points}}$

That's a hell of a lot of money! From my research, that's more than the fed spends on Veteran's benefits, Education, and the environment...**combined**. (source: <https://www.nationalpriorities.org/budget-basics/federal-budget-101/spending/>)

Lots of people get excited when they get a tax refund. But you need to remember – a tax refund means you gave the federal government more money than they deserved, and they kept it, interest – free, and they returned it to you, interest – free (often times, taking far longer than they should have to do so). Personally, I like to prepare my taxes so that, at worst, I owe the government a little bit of money; if I'm gonna keep extra money around, I'd rather put it in an interest – bearing account than give it to the feds to do...well, whatever it is they do with our money.

Quiz 7.

When we talked about measures of center in class, we focused on mean, median and mode. I'd like to look more deeply at the "mean" part of that trio in this quiz.

That mean (or average – the one where you add up the values of the data and then divide by how many data points there are) is, technically, called the "arithmetic mean". However, there are others! Their uses tend to be a bit too specific for this class (where we strive to be more far-reaching), but nonetheless, they have their place. Let's explore two of them, if you like!

Give a formula for each of the following two "means"! Assume in each that there are n data points. To give you an example of what I mean by a formula...for an arithmetic mean:

$$\text{Arithmetic Mean} = \frac{\sum x}{n}$$

- 1) **(2 points)** Geometric Mean.
- 2) **(2 points)** Harmonic Mean.
- 3) **(4 points)** An application of one of those...on February 5, 2017, I decided to take a road bike ride from Bend out to Prineville and back (same route both ways). On the way out, with a beautiful tailwind, I averaged 18.5 miles per hour. Of course, when I turned back to come home, I had to deal with an incredible headwind, which lowered my average speed for the whole trip to 15.3 miles per hour. I stopped only briefly to take the picture of my favorite Central Oregon courthouse at right (not significantly influential in the average calculations). What was my average speed on the return trip?
- 4) **(2 points)** Find an application of the other one!



Quiz 8.

Warmup extension!

So, in class a couple days ago, we looked at student debt. From the source listed in the footnote², we found that the average student debt was about \$29,500 (a grand total of about \$1.3 trillion spread over 44.1 million borrowers).

We then talked about those with **no** debt...were they included in this analysis, or were they **not**? I left that day thinking that they were...until I realized, as I walked up the hill to my office, that I was most likely wrong. ☺

I came to this realization by googling a few things. You can too!

1. **(2 points)** About how many Americans currently have at least associate's degrees? Be sure to give your source!

Since this number is far greater than 44.1 million, I figure that there are some folks fortunate enough to have no student loan debt³!

2. **(2 points)** From your answer to #1 (or more Googling), how many folks have no college debt? If you Googled, please list your source!
3. **(4 points)** (w) Use your answer from 2 and what we did in class on that warmup to recalculate the average from class. I mean, it's technically a **different** average, since we calculated "The average debt of those **with** debt", and I'm asking you to calculate "the average amount of debt of all college degree holders".
4. **(2 points)** How different are the two averages? Your choice as to how to measure the difference – but definitely **measure** it!
5. **(extra 1 point)** Find these two statistics! 1) The "median debt for those with debt" and the "Median debt for those with college degrees".

² <https://www.forbes.com/sites/zackfriedman/2017/02/21/student-loan-debt-statistics-2017/#3a340d525dab>

³ I'm assuming that people only have debt at the college level. . I sure hope to heck students aren't going into debt to finish **high school**.