7.3 Quadratic Patterns

- Recognize and describe a quadratic pattern.
- Use a quadratic pattern to predict a future event.
- Compare linear, quadratic, and exponential growth.

Study Tip

The word *quadratic* refers to terms of the second degree (or squared). You might remember from Algebra 1 that the quadratic formula is a formula for solving second degree equations.

Recognizing a Quadratic Pattern

A sequence of numbers has a **quadratic pattern** when its sequence of second differences is constant. Here is an example.

Terms: 1^2 2^2 3^2 4^2 5^2 6^2 7^2

1 4 9 16 25 36 49

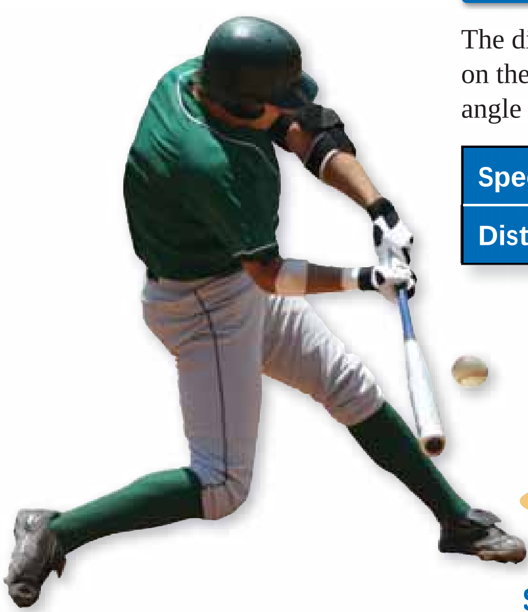
1st differences: 3 5 7 9 11 13 (49 − 36)

2nd differences: 2 2 2 2 2 (Constant)

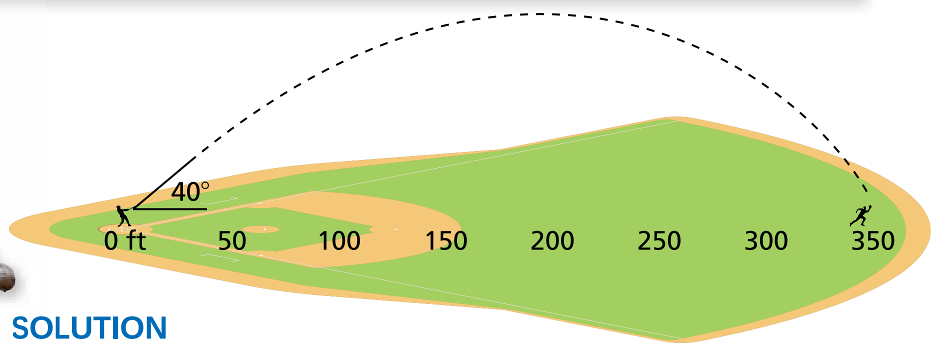
EXAMPLE 1 **Recognizing a Quadratic Pattern**

The distance a hit baseball travels depends on the angle at which it is hit and on the speed of the baseball. The table shows the distances a baseball hit at an angle of 40° travels at various speeds. Describe the pattern of the distances.

Speed (mph)	80	85	90	95	100	105	110	115
Distance (ft)	194	220	247	275	304	334	365	397



The distance a batter needs to hit a baseball to get a home run depends on the stadium. In many stadiums, the ball needs to travel 350 or more feet to be a home run.



SOLUTION

One way is to find the second differences of the distances.

194 220 247 275 304 334 365 397

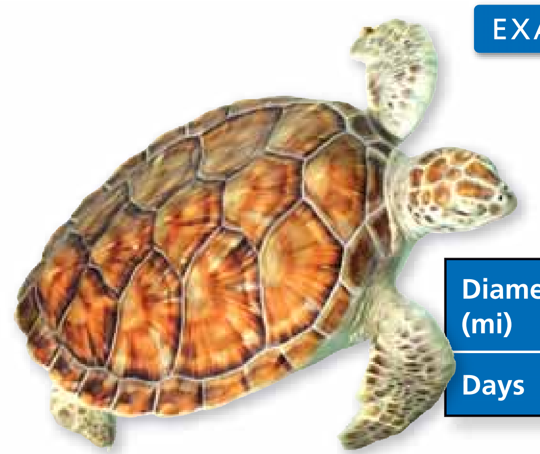
26 27 28 29 30 31 32

1 1 1 1 1 1 (Constant)

Because the second differences are constant, the pattern is quadratic.

Checkpoint Help at Math.andYOU.com

In Example 1, extend the pattern to find the distance the baseball travels when hit at an angle of 40° and a speed of 125 miles per hour.



The Institute for Marine Mammal Studies in Gulfport, Mississippi, reported that a large number of sea turtles were found dead along the Mississippi coast following the Deepwater Horizon oil spill of 2010.

EXAMPLE 2 Recognizing a Quadratic Pattern

The table shows the numbers of days an offshore oil well has been leaking and the diameters (in miles) of the oil spill. (a) Describe the pattern of the numbers of days. (b) Use a spreadsheet to graph the data and describe the graph.

Diameter (mi)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Days	0	1.5	6.0	13.5	24.0	37.5	54.0	73.5	96.0	121.5	150.0

SOLUTION

a. One way is to find the second differences of the numbers of days.

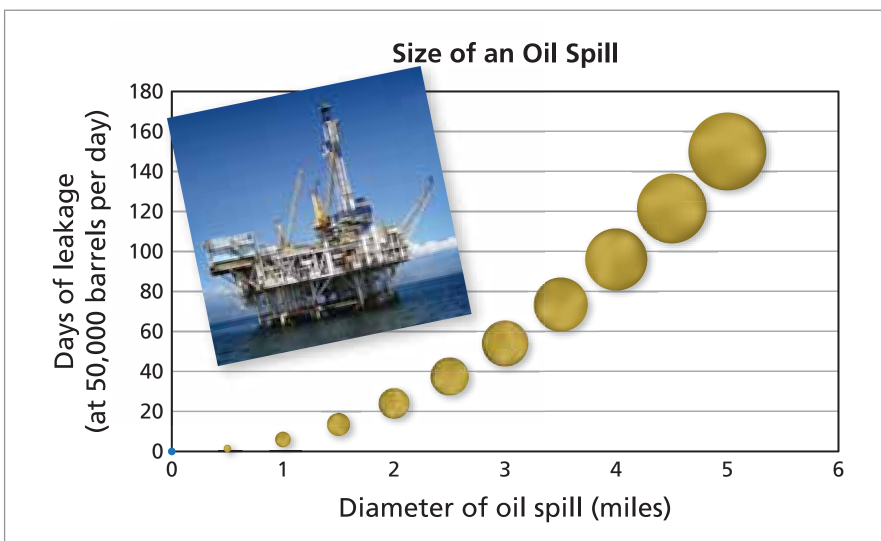
0 1.5 6.0 13.5 24.0 37.5 54.0 73.5 96.0 121.5 150.0

1.5 4.5 7.5 10.5 13.5 16.5 19.5 22.5 25.5 28.5

3 3 3 3 3 3 3 3 3 (Constant)

Because the second differences are constant, the pattern is quadratic.

b. The graph is a curve that looks something like exponential growth. However, it is not an exponential curve. In mathematics, this curve is called *parabolic*.



Checkpoint Help at Math.andYOU.com

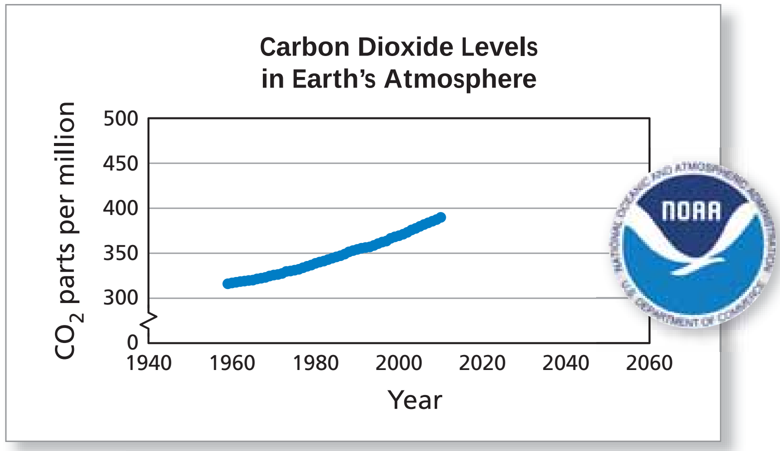
DATA Use a spreadsheet to make various graphs, including a scatter plot and a column graph, of the data in Example 1. Which type of graph do you think best shows the data? Explain your reasoning.

Using a Quadratic Pattern to Predict a Future Event

The Mauna Loa Observatory is an atmospheric research facility that has been collecting data related to atmospheric change since the 1950s. The observatory is part of the National Oceanic and Atmospheric Administration (NOAA).

EXAMPLE 3 Predicting a Future Event

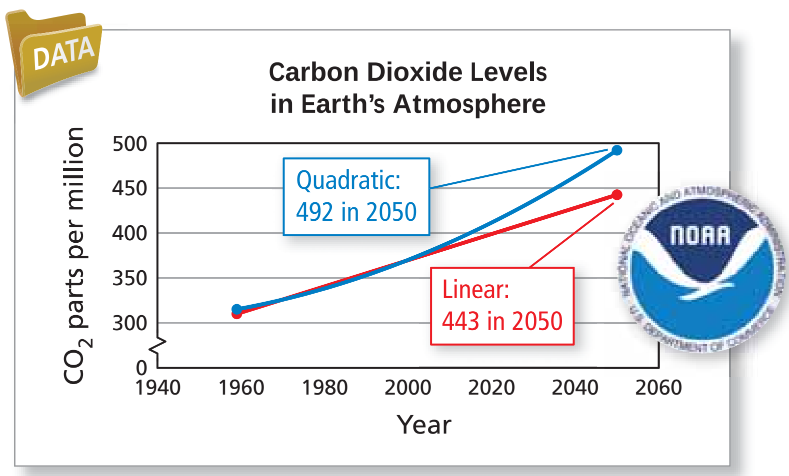
The graph shows the increasing levels of carbon dioxide in Earth's atmosphere. Use the graph to predict the level of carbon dioxide in 2050.



SOLUTION

The graph looks like it has a slight curve upward, which means that the rate of increase is increasing.

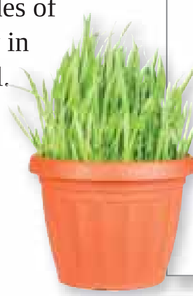
Using a linear regression program, the prediction for 2050 is 443 parts per million.

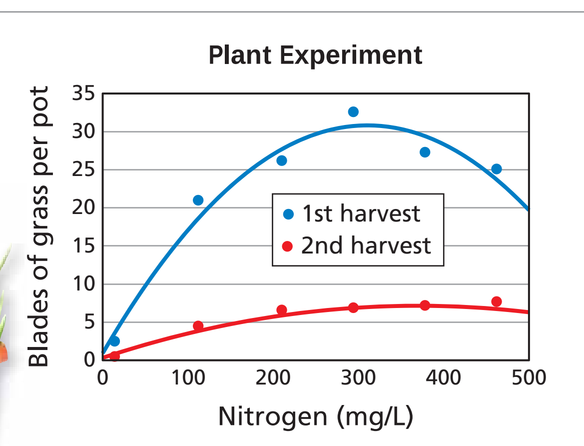


Using a quadratic regression program, the prediction for 2050 is 492 parts per million.

Checkpoint Help at Math.andYOU.com

The graph shows the results of a plant experiment with different levels of nitrogen in various pots of soil. The vertical axis measures the number of blades of grass that grew in each pot of soil. Describe the pattern and explain its meaning.





EXAMPLE 4 Describing Lift for Airplanes

For a given wing area, the lift of an airplane (or a bird) is proportional to the square of its speed. The table shows the lifts for a Boeing 737 airplane at various speeds.

Speed (mph)	0	75	150	225	300	375	450	525	600
Lift (1000s of lb)	0	25	100	225	400	625	900	1225	1600

The Boeing 737 is the most widely used commercial jet in the world. It represents more than 25% of the world's fleet of large commercial jet aircraft.

a. Is the pattern of the lifts quadratic? Why?

b. Sketch a graph to show how the lift increases as the speed increases.

SOLUTION

a. Begin by finding the second differences of the lifts.

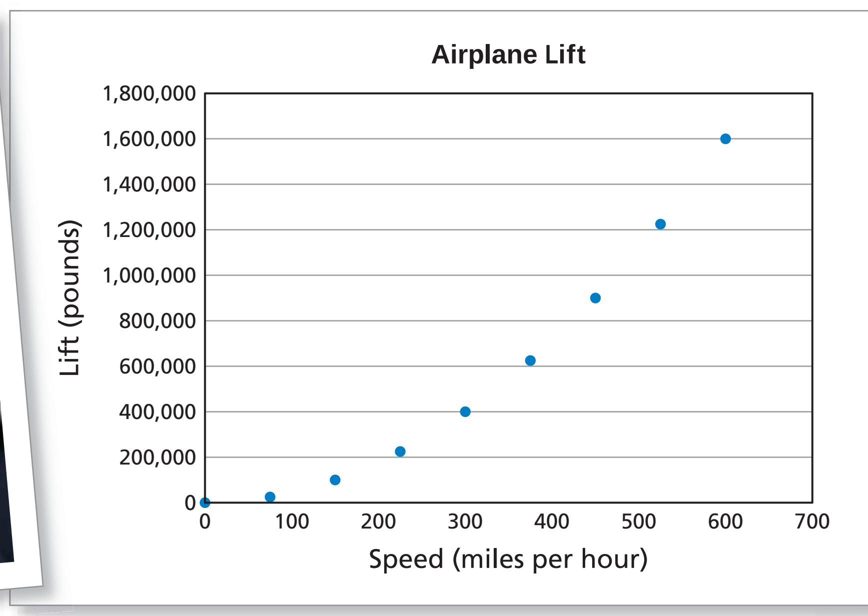
0 25 100 225 400 625 900 1225 1600

25 75 125 175 225 275 325 375

50 50 50 50 50 50 50 (Constant)

Because the second differences are constant, the pattern is quadratic.

b. Notice that as the speed increases, the lift increases quadratically.



✓ Checkpoint

Help at Math.andYOU.com

A Boeing 737 weighs about 100,000 pounds at takeoff.

c. Estimate how fast the plane must travel to get enough lift to take flight.

d. Explain why bigger planes need longer runways.

Comparing Linear, Exponential, and Quadratic Models

Earth's gravitational attraction was explained by Sir Isaac Newton's Law of Universal Gravitation. The law was published in Newton's *Principia* in 1687. It states that the force of attraction between two particles is directly proportional to the product of the masses of the two particles, and inversely proportional to the square of the distance between them.

EXAMPLE 5 Conducting an Experiment with Gravity

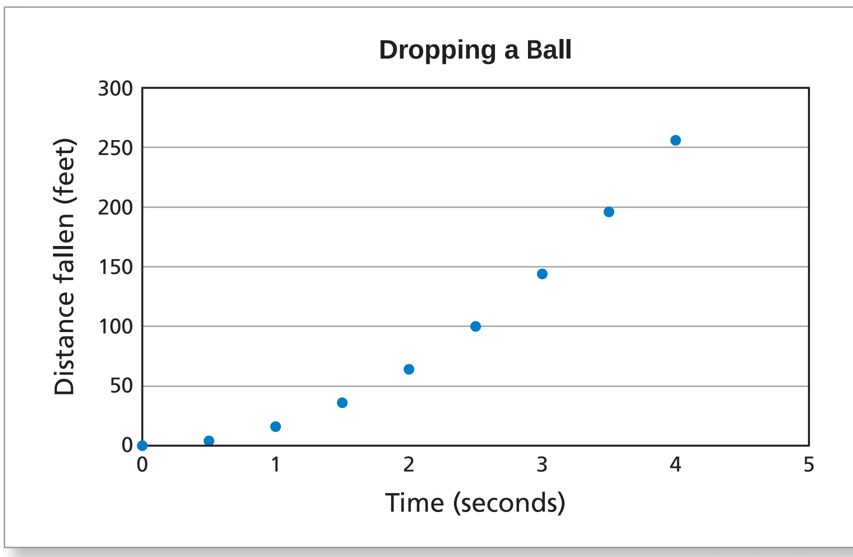
You conduct an experiment to determine the motion of a free-falling object. You drop a shot put ball from a height of 256 feet and measure the distance it has fallen at various times.

Time (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Distance (ft)	0	4	16	36	64	100	144	196	256

Is the pattern of the distances linear, exponential, quadratic, or none of these? Explain your reasoning.

SOLUTION

Begin by sketching a graph of the data.



- The pattern is *not* linear because the graph is not a line.
- The pattern is *not* exponential because the ratios of consecutive terms are not equal.
- The pattern *is* quadratic because the second differences are equal.

0 4 16 36 64 100 144 196 256

4 12 20 28 36 44 52 60

8 8 8 8 8 8 8 (Constant)

Checkpoint

Help at Math.andYOU.com

A classic problem in physics is determining the speed of an accelerating object. Estimate the speed of the falling shot put ball at the following times. Explain your reasoning.

a. 0 sec **b.** 1 sec **c.** 2 sec **d.** 3 sec **e.** 4 sec

EXAMPLE 6 Describing Muscle Strength

The muscle strength of a person's upper arm is related to its circumference. The greater the circumference, the greater the muscle strength, as indicated in the table.

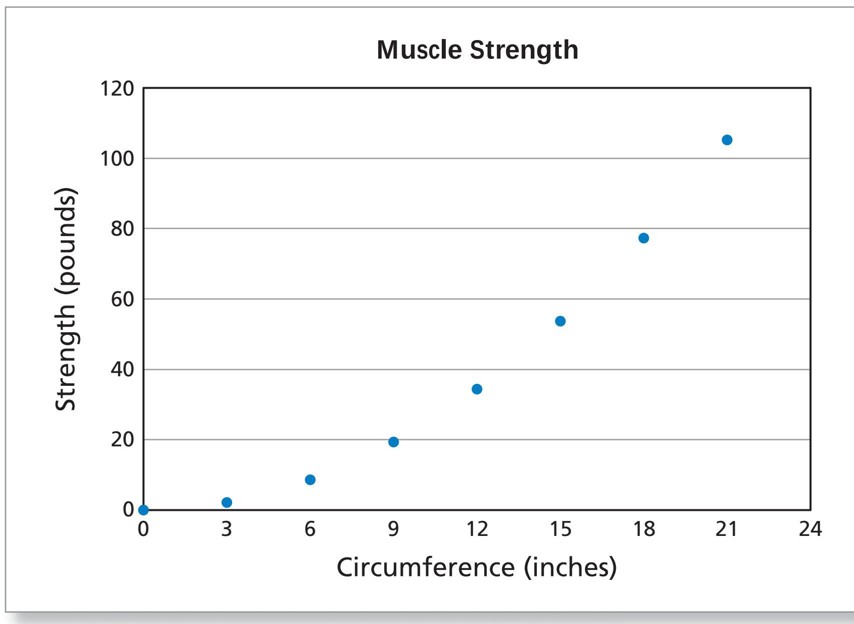
Circumference (in.)	0	3	6	9	12	15	18	21
Muscle strength (lb)	0	2.16	8.61	19.35	34.38	53.70	77.31	105.21

Is the pattern of the muscle strengths linear, exponential, quadratic, or none of these? Explain your reasoning.

SOLUTION

A typical upper arm circumference is about 12 inches for women and 13 inches for men.

Begin by sketching a graph of the data.



As in Example 5, the pattern is not linear or exponential. By calculating the second differences, you can see that the pattern is quadratic.

0 2.16 8.61 19.35 34.38 53.70 77.31 105.21

2.16 6.45 10.74 15.03 19.32 23.61 27.90

4.29 4.29 4.29 4.29 4.29 4.29 (Constant)

Checkpoint

Help at Math.andYOU.com

Example 6 shows that the muscle strength of a person's upper arm is proportional to the square of its circumference. Which of the following are also true? Explain your reasoning.

a. Muscle strength is proportional to the diameter of the muscle.

b. Muscle strength is proportional to the square of the diameter of the muscle.

c. Muscle strength is proportional to the cross-sectional area of the muscle.

7.3 Exercises

DATA **Football** **In Exercises 1–3, describe the pattern in the table.** ***(See Examples 1 and 2.)***

1. The table shows the heights of a football at various times after a punt.

Time (sec)	0	0.5	1	1.5	2	2.5	3
Height (ft)	3	34	57	72	79	78	69

2. The table shows the distances gained by a running back after various numbers of rushing attempts.

Rushing attempts	0	3	6	9	12	15	18
Distance (yd)	0	12.6	25.2	37.8	50.4	63	75.6

3. The table shows the heights of a football at various times after a field goal attempt.

Time (sec)	0	0.5	1	1.5	2	2.5	3
Height (ft)	0	21	34	39	36	25	6

4. **Punt** In Exercise 1, extend the pattern to find the height of the football after 4 seconds. *(See Example 1.)*

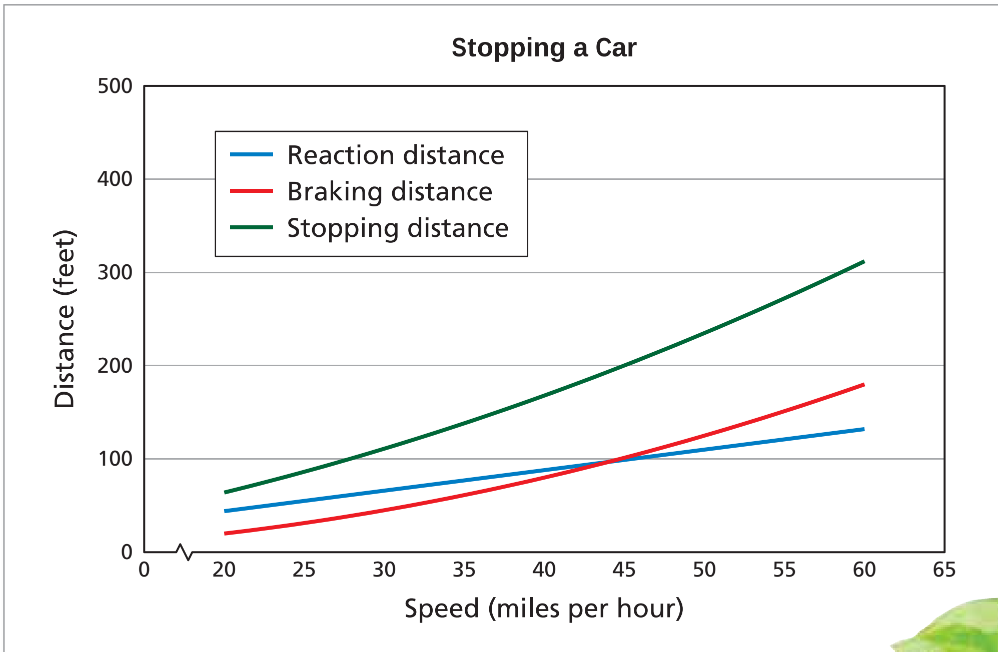
DATA **5.** **Passing a Football** The table shows the heights of a football at various times after a quarterback passes it to a receiver. Use a spreadsheet to graph the data. Describe the graph. *(See Example 2.)*

Time (sec)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
Height (ft)	6	15	22	27	30	31	30	27	22	15	6

DATA **6.** **Graph** Use the graph in Exercise 5 to determine how long the height of the football increases.

Stopping a Car **In Exercises 7–10, use the graph and the information below.** ***(See Example 3.)***

Assuming proper operation of the brakes on a vehicle, the minimum stopping distance is the sum of the reaction distance and the braking distance. The reaction distance is the distance the car travels *before* the brakes are applied. The braking distance is the distance a car travels *after* the brakes are applied but *before* the car stops. A reaction time of 1.5 seconds is used in the graph.



7. Does the graph of the stopping distance appear to be linear or quadratic? Explain your reasoning.

8. Does the graph of the reaction distance appear to be linear or quadratic? Explain your reasoning.

9. Use the graph to predict the stopping distance at 90 miles per hour.

10. The braking distance at 35 miles per hour is about 60 feet. Does this mean that the braking distance at 70 miles per hour is about 120 feet? Explain.

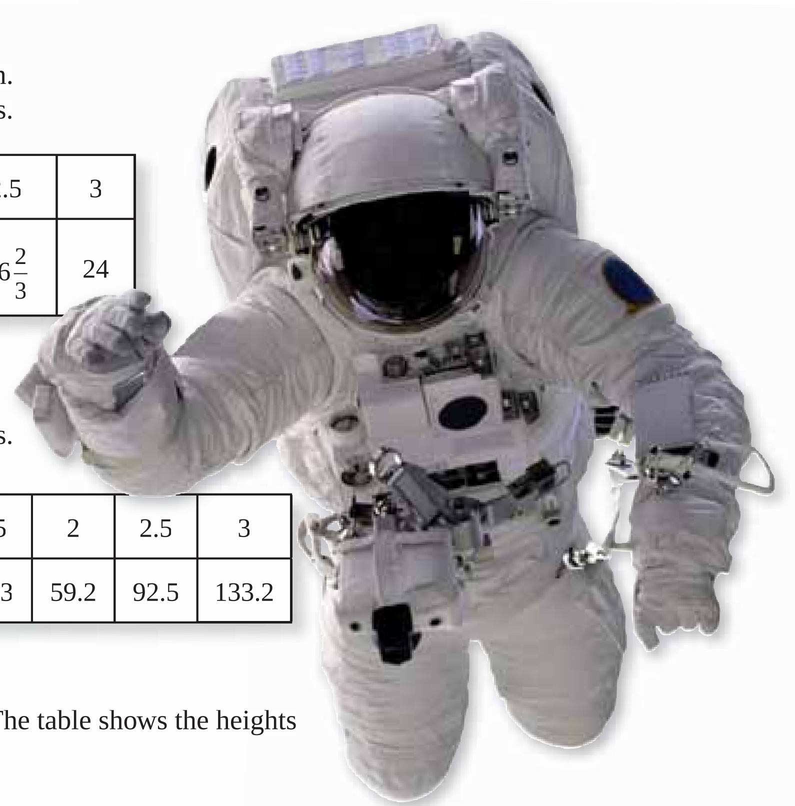
DATA **Slippery Road** **The braking distance of a car depends on the friction between the tires and the road. The table shows the braking distance for a car on a slippery road at various speeds. In Exercises 11 and 12, use the table.** ***(See Example 4.)***

Speed (mph)	20	30	40	50	60	70	80
Distance (ft)	40	90	160	250	360	490	640

11. Is the pattern quadratic? Explain.

12. Graph the data in the table. Compare this graph to the graph above.

Gravity In Exercises 13–16, determine whether the pattern in the table is linear, exponential, quadratic, or none of these. Explain your reasoning. ***(See Examples 5 and 6.)***



13. An object is dropped from a height of 50 feet on the moon. The table shows the distances it has fallen at various times.

Time (sec)	0	0.5	1	1.5	2	2.5	3
Distance (ft)	0	$\frac{2}{3}$	$2\frac{2}{3}$	6	$10\frac{2}{3}$	$16\frac{2}{3}$	24

14. An object is dropped from a height of 150 feet on Venus. The table shows the distances it has fallen at various times.

Time (sec)	0	0.5	1	1.5	2	2.5	3
Distance (ft)	0	3.7	14.8	33.3	59.2	92.5	133.2

15. An object is dropped from a height of 300 feet on Mars. The table shows the heights of the object at various times.

Time (sec)	0	1	2	3	4	5	6
Height (ft)	300	293.8	275.2	244.2	200.8	145	76.8

16. An object is dropped from a height of 1600 feet on Jupiter. The table shows the heights of the object at various times.

Time (sec)	0	1	2	3	4	5	6
Height (ft)	1600	1556.8	1427.2	1211.2	908.8	520	44.8

17. Sign of Second Differences Graph the data in Exercises 14 and 15 on the same coordinate plane. Compare the graphs. What appears to be the relationship between the sign of the second differences and the corresponding graph?

18. Moon The moon's gravitational force is much less than that of Earth. Use the table in Exercise 13 and the table in Example 5 on page 328 to estimate how many times stronger Earth's gravitational force is than the moon's gravitational force. Explain your reasoning.

Extending Concepts

Business **Data from real-world applications rarely match a linear, exponential, or quadratic model perfectly. In Exercises 19–22, the table shows data from a business application. Determine whether a linear, exponential, or quadratic model *best* represents the data in the table. Explain your reasoning.**

19. The table shows the revenue for selling various units.

Units sold	0	40	80	120	160	200
Revenue	\$0	\$186.30	\$372.45	\$558.38	\$744.24	\$930.15

20. The table shows the total cost for producing various units.

Units produced	0	40	80	120	160	200
Total cost	\$500.00	\$572.05	\$627.98	\$668.03	\$692.10	\$700.12

21. The table shows the profit from selling various units.

Units sold	0	40	80	120	160	200
Profit	−\$500.00	−\$385.75	−\$255.53	−\$109.65	\$52.14	\$230.03

22. The table shows the stock price of a company for various years.

Year	2007	2008	2009	2010	2011	2012
Stock price	\$21.56	\$23.68	\$26.08	\$28.62	\$31.62	\$34.79

Activity **Fold a rectangular piece of paper in half. Open the paper and record the number of folds and the number of sections created. Repeat this process four times and increase the number of folds by one each time. In Exercises 23–26, use your results.**

2 folds
4 sections

23. Complete the table.

Folds	1	2	3	4	5
Sections					

24. Graph the data in Exercise 23. Determine whether the pattern is linear, exponential, or quadratic.

25. Write a formula for the model that represents the data.

26. How many sections are created after eight folds?

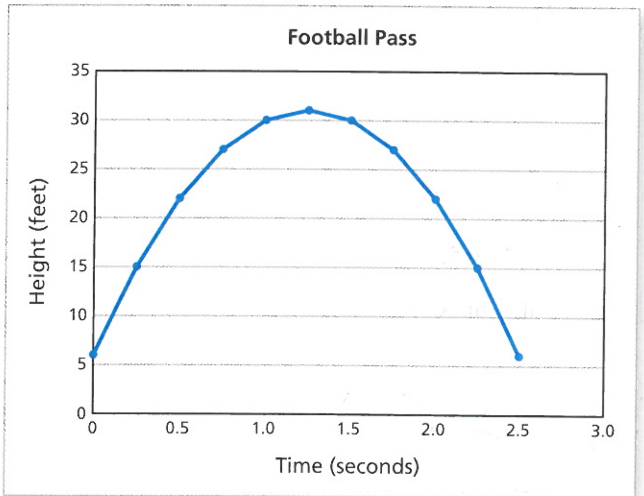
5. 2025

7. Set B; $\frac{200}{1.12^5} \approx 113$, but $200 - (5 \times 24) = 80$.

Section 7.3 *(page 330)*

1. The second differences are constant (-8). The pattern is quadratic.

3. The second differences are constant (-8). The pattern is quadratic.

5. 

The pattern is quadratic.
The graph is a downward U-shaped curve.

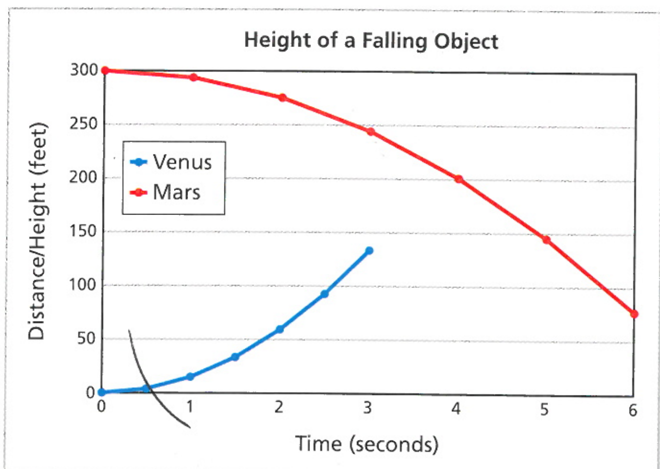
7. Quadratic; The graph is curving upward.

9. About 600 ft

11. Yes; The second differences are constant (20).

13. Quadratic; The second differences are constant $\left(1\frac{1}{3}\right)$.

15. Quadratic; The second differences are constant (-12.4).

17. 

The graph of the data from Venus curves upward. The graph of the data from Mars curves downward. It appears that if the second differences are positive, then the graph curves upward, and if the second differences are negative, then the graph curves downward.

19. Linear; The first differences are about 186.

21. Quadratic; The second differences are about 16.

23.

Folds	1	2	3	4	5
Sections	2	4	8	16	32

25. $S = 2^n$

Section 7.4 *(page 340)*

1. The number of petals on a daisy is usually a Fibonacci number.