## A Quadratic Equation Application

Find the dimensions of a STOP sign with a total area of $900 \mathrm{in}^{2}$.
Each edge is equal. We denote the edge by S. x denotes the horizontal or vertical distance cut out of each corner. Note: $\mathrm{x} \neq \mathrm{S}$. Why? Recognize that each corner is a right triangle and therefore must obey the Pythagorean Theorem. $x^{2}+x^{2}=S^{2}$. This leads to $2 x^{2}=S^{2}$ Why? Solving for $S$ gives us $\mathrm{S}=\mathrm{x} \sqrt{2}$. Why? We can now find x by solving the following equation:

$$
\text { (4-triangles, 4-rectangles and one square in the center } \left.=900 \mathrm{in}^{2}\right)
$$


turn the sentence into an equation
substitute out $S$ to get only ' $x$ '

$$
\begin{gathered}
4 x^{2}+4 \sqrt{2} x^{2}=900 \\
(4+4 \sqrt{2}) x^{2}=900 \\
x^{2}=\frac{900}{4+4 \sqrt{2}} x=\sqrt{\frac{900}{4+4 \sqrt{2}}}=\sqrt{\frac{225}{1+\sqrt{2}}} \\
S=\sqrt{2} \sqrt{\frac{225}{1+\sqrt{2}}} \rightarrow S=\sqrt{\frac{450}{1+\sqrt{2}}}
\end{gathered}
$$

combine like terms and then factor $x^{2}$
divide both sides by $(4+4 \sqrt{2})$
simplify
substitute back to get $S, S \approx 13.65^{\prime \prime}$

Another approach to solving the above problem would be to begin with the circumscribing square and subtract the 4 corner triangles to obtain the 900 sq -in of area: (surrounding square -4 -triangles $=900 \mathrm{in}^{2}$ )

$$
(x+S+x)^{2}-4\left[(1 / 2) x^{2}\right]=900 \quad S=\sqrt{2} x \text { and simplifying yields: }
$$

$$
(2 x+\sqrt{2} x)^{2}-2 x^{2}=900
$$

This requires we multiply (expression) $\times$ (expression) which we shall discuss in the next section. There are numerous situations where we might want to multiply a pair of expressions. The last example presented us with the alternate problem formation leading to the need to solve $(2 x+\sqrt{2} x)^{2}-2 x^{2}=900$.

Although we can approximate $\sqrt{2}$ and combine the two terms here we shall develop a general process for multiplying (expression) $\times$ (expression). This will allow us to solve a wider array of problems.

