Beauty and the Greek *Franz Helfenstein* Name

It's interesting that three of the greatest ancient philosophers were all alive during the same time period: **Pythagoras** (569? – 475? BC), **Buddha** (563? – 483? BC) and **Confucius** (551? – 479? BC).

Pythagoras is most famous for the *Pythagorean Theorem*, $a^2 + b^2 = c^2$, which refers to the lengths of the sides of a right triangle. At that time, at least among the Greeks, there was a belief that beauty and mathematics were intertwined. The integers were more appealing than the rational numbers and perfect squares were extra special. Pythagoras and the *Pythagoreans* (as his followers were called) knew that a 3-4-5 triangle was a right triangle (why?) and thus formed a *Pythagorean Triple* (a set of three numbers which obey $a^2 + b^2 = c^2$). Since 3, 4, 5 are also consecutive integers this was indeed special.

The *Pythagoreans* wanted to find more integer Pythagorean Triples. Since the Pythagoreans were somewhat secretive little is know of the specifics but they surely were aware of the following result which dates from the Babylonians, circa 1900 BC.

> Given any two integers p & q a Pythagorean Triple is formed from: $p^2 - q^2$, $2pq$ and $p^2 + q^2$

For Example: $p = 2, q = 1 \rightarrow p^2 - q^2 = 2^2 - 1^2 = \boxed{3}$, $2pq = 2 \cdot 2 \cdot 1 = \boxed{4}$ and $p^2 + q^2 = 2^2 + 1^2 = \boxed{5}$.

1) Pick 3 other random integers and form some Pythagorean Triples.

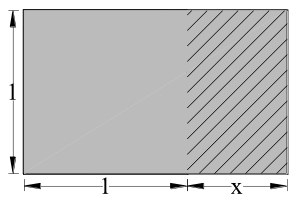
	a	b	c
(i)			
(ii)			
(iii)			

2) Using **a** = $p^2 - q^2$, **b** = $2pq$ and **c** = $p^2 + q^2$ prove algebraically that $a^2 + b^2 = c^2$.

Start with $a^2 + b^2 = c^2$. Use substitution to replace a, b & c with p's & q's and simplify to show both sides are equal.

3) In their quest to quantify beauty the Greeks concluded that a rectangle which obeyed the criteria of a *Golden Ratio* was the most perfect shape. In many Greek sculptures, the face, the eyes the nose, the mouth etc all fit within a *Golden Rectangle*. Similarly, many Greek monuments incorporate the Golden Ratio.

Any rectangle where a square can be cut off to leave a rectangle of similar ratio to the original ratio is called a Golden Rectangle and obeys the Golden Ratio.



In other words, we have a Golden Ratio here if 1 : x+1 is the same as x : 1. *Why?*

To find 'x' and hence determine the Golden Ratio we must solve the equation $\frac{1}{x+1} = \frac{x}{1}$. *Why?*

4) To solve the above equation, the Greeks needed to consider non-perfect square roots which was quite a radical concept at the time. Consequently, numbers such as $\sqrt{5}$ (which cannot be expressed as a fraction) are called *irrational numbers*. Solve the equation and find the <u>precise</u> Golden Ratio.

5) Round 'x' to the <u>nearest tenth</u> and find an equivalent ratio with <u>integers</u>. How does this compare with the common size of Post Cards, Snap Shots, paper: (3 × 5), (4 × 6), (8 ½ × 11)?