Lab 1 **Name**

Part 1: Round and Round

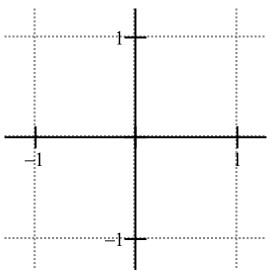
Consider the circle $x^2 + y^2 = 4$. We want to construct parametric curves that will trace this circle in different ways. Start with a parametric function of the form:

$$x = a_1 \cos(a_2 t)$$
$$y = a_3 \sin(a_4 t)$$
$$0 \le t \le 2\pi \qquad \text{where } a_n \in \mathbb{R}$$

This means that the a_n's can be any real number.

(1) Find a_1 and a_3 so that every point (x,y) lies on the circle $x^2 + y^2 = 4$.

(2) Describe the motion of the particle if $a_2 = a_4 = 1$. Use a <u>sketch</u>, and label the start and end points. Use a <u>sentence</u> to describe the motion.



(3) Describe the motion of the particle if $a_2 = a_4 = -1$. Use a <u>sentence</u> to describe the motion.

(4) Describe the motion of the particle if $a_2 = a_4 = 2$. Use a <u>sentence</u> to describe the motion.

(5) Find a_2 and a_4 so that the particle goes counterclockwise five times around the circle, starting at (2,0).

(6) Find a_2 and a_4 so that the particle goes clockwise three times around the circle, starting at (2,0).

Part 2: Lissajous Figures

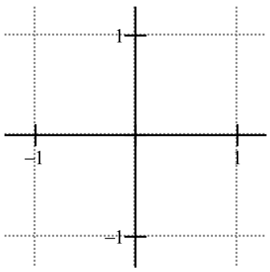
Again, start with a parametric function of the form:

$$x = a_1 \cos(a_2 t)$$
$$y = a_3 \sin(a_4 t)$$
$$0 \le t \le 2\pi \ (or\ \infty) \qquad \text{where } a_n \in \mathbb{R}$$

These curves are called Lissajous figures, and are used in electrical engineering to see if two signals are "in sync". They can also be used in music to show whether a musical interval is in tune.

For the following problems, let $a_1 = a_3 = 1$ so that we can investigate the effect of a_2 and a_4 on the function.

(1) Graph x = cos (1t), y = sin (2t) on your calculator, graphing it below, and labelling the start and end points. Use a sentence to describe the motion.

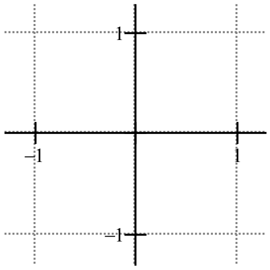


(2) Graph x = cos (2t), y = sin (4t) on your calculator. How is it the same as the previous graph, and how is it different? Use a sentence to describe the motion.

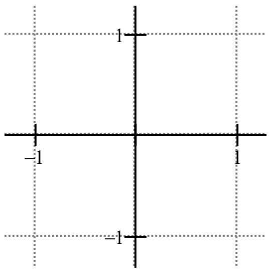
(3) Graph x = cos (3t), y = sin (6t) on your calculator. How is it the same as the previous two graphs, and how is it different? Use a sentence to describe the motion.

(4) Fix a_2 = 1, and graph the funcitons with a_4 = 1, 2, 3.

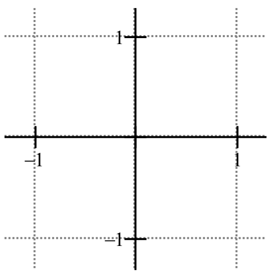
a_4 = 1



a_4 = 2

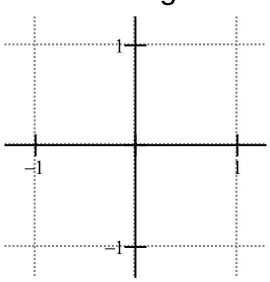


a_4 = 3

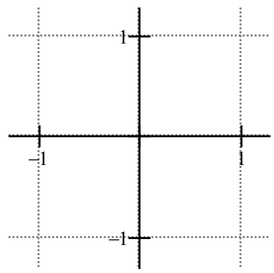
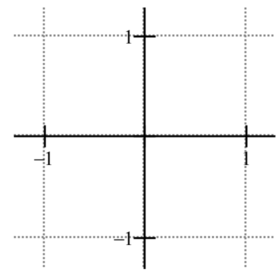
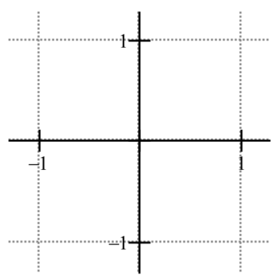
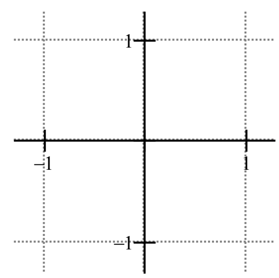


Describe what happens as a_4 increases. Use a <u>sentence</u>.

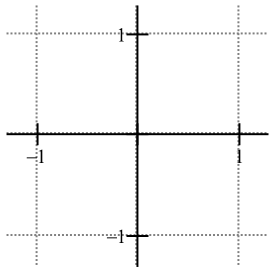
Predict what the figure will look like if a_2 = 1 and a_4 = 5. Include a careful sketch.



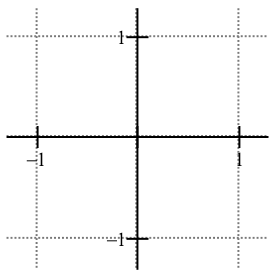
(5) Now fix a_4 = 1, and look (on your calculator) at graphs for a_2 = 1, 2, 3, 4. What happens as a_2 increases? Describe the effect carefully, using graphs to illustrate.

<table>
<tr><th>a_2 is odd</th><th>a_2 is even</th></tr>
<tr><td>a_2 = 1
</td><td>a_2 = 2
</td></tr>
<tr><td>a_2 = 3
</td><td>a_2 = 4
</td></tr>
<tr><td>If a_2 is odd, you get ...</td><td>If a_2 is even, you get ...</td></tr>
</table>

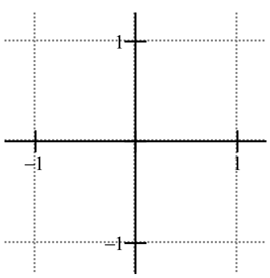
(6) Now (on your calculator) try a_2 = 1, and a_4 = $\sqrt{2}$. Try Tmax = 2π, then 4π, 10π, and 20π, and so on. What would happen if Tmax was infinity? Sketch the final result below.



(7) Now try a_2 = $\sqrt{5}$ and a_4 = $\sqrt{2}$, again letting t get very large. Sketch the final result below.

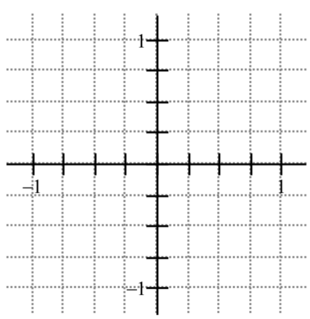


(8) Now try a_2 = $\sqrt{18}$ and a_4 = $\sqrt{2}$. Graph it below. Explain why this one is different from the two previous examples. What would be the simplest possible values of a_2 and a_4 that would give the same final picture?



(9) I've said that Lissajou figures can be used to check whether musical intervals are in tune, whether they will sound good or bad. What would you guess would have to be the relationship between a_2 and a_4 for a musical interval to sound good? Start by thinking about the examples on the previous page – which would you guess would sound good, and which would sound bad? Alternately, which pictures do you think show that electrical signals are in sync, and which pictures show that electrical signals are not in sync?

(10) The figure with a_4 = 1, and a_2 = 2 has a familiar geometric shape – in fact, it's one of the conic sections. Graph it below. Use a trig identity[1] for cos(2t) to explain why the figure looks the way it does.



[1] Look up the trig identities in the front of your book. Use substitution to convert the parametric equations to one Cartesian (x-y) equation.