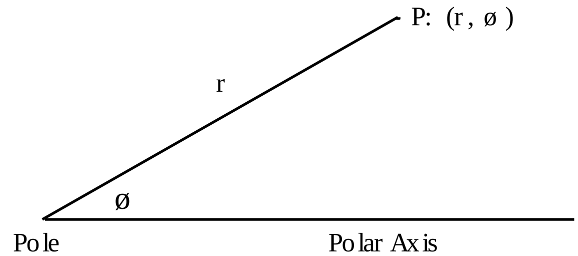
As we have seen, a rectangular coordinate system is not the only point of view to represent functions. We have explored *parametric equations* as another alternative. This lab is intended to expand on a third point of view: *polar coordinates*.

Goals: What should you learn from this activity?

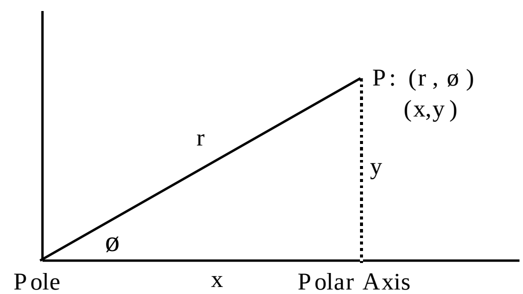
a) To understand the concept of polar coordinates.
b) To be able to convert from polar to rectangular coordinates.
c) To be able to recognize some classes of functions expressed in polar coordinates.

Investigation 1: Expressing points in polar coordinates.

The polar coordinates of a point are determined by the point's distance from the **pole** (origin) and the angle between a **polar axis** and a ray connecting the point and the pole. Angles are measured positively in the *counterclockwise* direction.



If an xy–coordinate system is superimposed on the polar coordinate system, with the origins coinciding, then it is possible to determine a relationship between the rectangular coordinates of P, (x,y), and the polar coordinates of P, (r,θ).



Based on the figures above, fill in the following boxes:

x =	r^2=
y =	tan θ =
Note: $\tan^{-1}$ is quadrant dependent!	

1. Using these results, convert the following *rectangular* points to polar points:

(x, y)	(0,0)	(1,0)	(0,1)	(1,1)	(–1,2)	(–1,–1)
(r, θ)						

2. Using these results, convert the following *polar* points to rectangular points angles are always measured in radians:

(r, θ)	(0,0)	(1,0)	(1,π)	(–1,0)	(–1,π/4)	(1,π/2)	(1,1)
(x, y)							

Investigation 2: Graphs

The **graph of a polar equation** $r = f(\theta)$ consists of all points, P, that have at least one polar representation (r, θ) whose coordinates satisfy the equation.

1. Describe the graphs of the following equations. Use the TI–Calculator set in Polar mode.

   a. $r = a$ $\quad 0 \le \theta \le 2\pi$ $\quad$ for various choices of a

   b. $\theta = \pi/3$

   c. $r = a\,\theta$ $\quad 0 \le \theta \le 2\pi$ $\quad$ for various choices of a

2. The equation of a line is $ax + by = c$. Convert this to a polar equation and write the resulting equation in the form $r = f(\theta)$. Graph $r = f(\theta)$ and compare with $ax + by = c$.

3. We know that $y = m\,x^2$ represents a parabola. Convert this to a polar equation and write the resulting equation in the form $r = f(\theta)$. Graph $r = f(\theta)$ and compare with $y = m\,x^2$.

4. We know that $(x - 1)^2 + y^2 = 1$ represents a circle of radius one, centered at (1, 0). Convert this to a polar equation and write the resulting equation in the form $r = f(\theta)$. Graph $r = f(\theta)$ and compare with $(x - 1)^2 + y^2 = 1$.

5. How would you represent the graph of a circle centered at (a, 0) with radius a?

6. How would you represent the graph of a circle centered at (0, a) with radius a?

Investigation 3: Generalizing graphs of circles

1. Graph the equation $r = a \sin \theta + b \cos \theta$ for various choices of a and b. Describe the resulting curves.

2. Convert the above equation to rectangular coordinates by performing the following steps:

    a. Multiply both sides of the equation by r.
    b. Substitute rectangular coordinates for r and θ into the equation.
    c. *Complete the square* in both the x and y terms.
    d. Verify that the resulting equation is the equation of a circle.

3. Explain the effect of different choices of a and b on the shape of the circle.

Investigation 4: Other classes of polar graphs

Examine the following classes of graphs. Use a variety of values for a and b. Summarize your results.

1. *Cardioids and Limaçons*

   $r = a \pm b \sin \theta$ $\quad\quad$ $r = a \pm b \cos \theta$

2. *Roses*

   $r = a \cos n\theta$ $\quad\quad$ $r = a \sin n\theta$

3. *Spirals*

   $r = \theta$ $\quad\quad$ $r = 1/\theta$ $\quad\quad$ $r = e^{\theta}$

4. *Odds 'n Ends*

   $r^2 = \sin 2\theta$ $\quad\quad$ $r^2 = 4 \cos 2\theta$

   $r = 40 + 5 \sin(4\theta)$