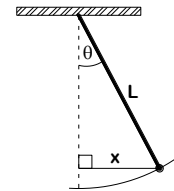


## Modeling a Pendulum's Motion

Name(s): \_\_\_\_\_

The horizontal (or vertical) motion of a pendulum may be modeled by  $x(t) = A \sin(bt + c)$ . We want to understand the relation between  $A$ ,  $b$  and  $c$  and the pendulum.

- 1) Measure out 2m of line and start the pendulum gently swinging. Measure the time for 10 cycles and divide by 10 to get the period ( $T$ ) in seconds. ( $T$  = time for one complete cycle)  
Note: the frequency is  $1/T$  or  $f = 1/T$ . Why?



$T =$  \_\_\_\_\_ sec       $f =$  \_\_\_\_\_ cycles/sec

- 2) It has long been known that a swinging pendulum's frequency depends on the length of the pendulum with  $f(L) = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$  cycles/sec. Using  $g = 9.8 \text{ m/sec}^2$  compute  $f$  and compare with your real data.

using  $f(L)$   $f =$  \_\_\_\_\_ sec       $T(L) =$  \_\_\_\_\_      using  $T(L)$   $T =$  \_\_\_\_\_ hz

- 3) Explain how we determine  $b$ ?  $b$  depends on  $L$  but it is not the same as  $f$ . What is  $b(L)$ ?

$b(L) =$  \_\_\_\_\_       $b(2m) =$  \_\_\_\_\_

- 4) Suppose we start the pendulum by first moving it 20 cm off center. That is,  $x(0) = 20 \text{ cm}$

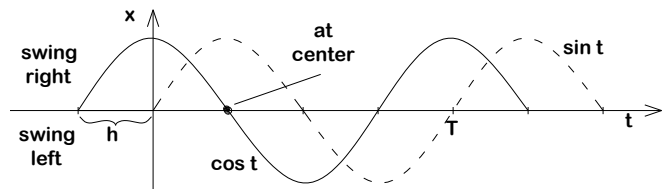
(a) How do we determine  $A$ ? What is it?

- 5)  $h$  depends on  $T$ . What is  $h(T)$ ?

$h(T) =$  \_\_\_\_\_  $\approx$  \_\_\_\_\_

- 6)  $c$  depends on  $h$ . What is  $c(h)$ ?

$c(h) =$  \_\_\_\_\_  $\approx$  \_\_\_\_\_



- 7) Now restart the pendulum by first moving it 30cm off center. That is,  $x(0) = 30\text{cm}$ . After 5, 10, 15 and 20 swings, record the maximum position of the horizontal displacement.

$x(0) = 30 \text{ cm}$        $x(5T) =$  \_\_\_\_\_       $x(10T) =$  \_\_\_\_\_       $x(15T) =$  \_\_\_\_\_       $x(20T) =$  \_\_\_\_\_

Dampened pendulum motion is modeled by  $x(t) = A e^{-kt} \sin(bt + c)$ . We want to find 'k' 2 ways.

- 8) Use  $x(0)$  and  $x(5T)$  to compute 'k' algebraically. You must show your work.

- 9) Use all 5 data points and run exponential regression to find 'k'. From TI regression:  $y = ab^x$  Use change of bases to find 'k'. Solve  $b^x \rightarrow e^{-kx}$  for k. You must show your work. Hint: Take ln of both sides.