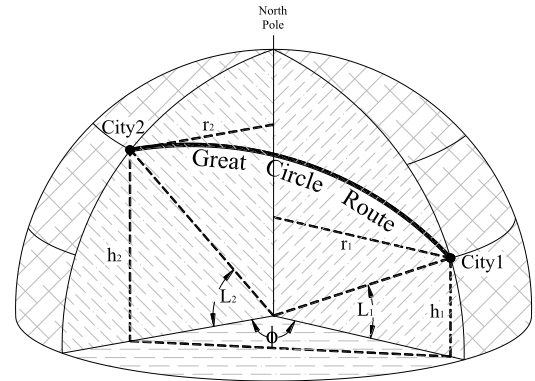


The Great Circle Route is the shortest distance between two points on the globe's surface. Given the Longitude (M) and Latitude (L) of two such locations, the arc distance (s) can be found using this two stage formula.

$$\cos(\theta) = \sin L_1 \sin L_2 + \cos L_1 \cos L_2 \cos(M_2 - M_1)$$

$$s = 2\pi R_E \cdot \theta^\circ / 360^\circ; \quad R_E \approx 3,960 \text{ mi}$$

where θ is the angle created in the plane formed by the two points on the Earth's surface and the Earth's center.



Our task is to verify these formulas for the *Great Circle Route* distance through their derivation and to describe that derivation (in detail) in a report. The report should clearly show the derivation of both stages of the formula. The derivation must be written in **general terms**. That is, we are finding θ as a function of L_1 , L_2 , M_1 , M_2 , R .

For demonstration purposes the following example should be included after the general case formula has been verified: Find the distance from Bend ($44^\circ 4' \text{ N}$, $121^\circ 18' \text{ W}$) to Sydney, Australia (34° S , 151° E). Under no circumstances should you be deriving the general case function using these values. The derivation must be written in **general terms**.

You may work together as a team of 2 and turn in one report or work individually. If you work as a team, you must keep a journal and include a peer assessment with your report. If you work individually, the journal is optional. Either way, you will be graded on clarity, organization, accuracy and timeliness. This is just as much about writing/explaining math as doing calculations.

This project is **optional**. Although it is optional, I highly encourage you to do it. It will solidify your understanding of trigonometry, give you invaluable experience working on a rather intricate problem and I believe you will find it personally rewarding to find that you are capable of such a task. For those completing the project, I will reward you with the option of forgoing your final exam or replacing any other exam.

One of the greatest difficulties here arises in trying to draw diagrams in a 2-D plane that accurately depict 3-D geometry. So I have provided you with a paper cutout to make a 3-D model that will be useful in seeing those relationships. You'll need to work with the Pythagorean Theorem, similar triangles, arc lengths and trigonometry.

Key steps:

- 1) Cut out and build the paper model to help you see the imbedded triangles.
- 2) Work backwards to find s as a function of R and θ .
- 3) Although θ generates the Great Circle arc it is also part of a sector and a **non-right triangle**.
- 4) Notice that C is also part of a **right triangle** with sides h , C and D where $h = h_2 - h_1$.
- 5) Notice that h_1 and h_2 are each part of a **right triangle** with an included angle related to latitude.
- 6) Notice that D is part of two different triangles. A right triangle previous referred to in (4) and another one, a **non-right triangle**. This non-right triangle is positioned horizontally in the plane of the equator with sides r_1 , D , r_2 and angle ϕ .
- 7) Notice that r_1 (or r_2) forms the base of a **right triangle** as mentioned in (6).
- 8) Notice that ϕ is the meridian angle from City₁ to City₂. That is, ϕ can be found using Longitudes.
- 9) Now put this all together in reverse order to obtain the derivation.

OVERALL FORMAT- Layout/Organization/Presentation

35 pts *Typed with clearly readable font or neatly handwritten. Uses title and other clarifying headings. Layout and information organization/presentation flows for easy readability.*

0 pts *Readability is significantly hindered by font type, handwriting, layout and/or information presentation.*

- | | |
|---|---|
| <input type="checkbox"/> Title | <input type="checkbox"/> Clear Well Placed Diagram(s) |
| <input type="checkbox"/> Appropriate use of Headings | <input type="checkbox"/> Appropriate / Clear Notation |
| <input type="checkbox"/> Section Breaks Apparent | <input type="checkbox"/> Appropriate Layout of Multiple Steps |
| <input type="checkbox"/> Appropriate use of White Space | <input type="checkbox"/> Spelling / Grammar |
| <input type="checkbox"/> Easy on the Eyes / Overall Readability | <input type="checkbox"/> Appropriate for Audience |

INTRODUCTION

15 pts *Clear, easy to read. Provides motivation, includes clear problem statement and flows naturally into general procedure. Makes the reader want to continue reading. Diagrams enhance Introduction.*

0 pts *Quite confusing overall. Motivation missing or unclear. Problem statement missing, awkward or incorrect. Makes the reader want to tear out their hair (even if they are already bald).*

- | | |
|---|---|
| <input type="checkbox"/> Human Interest Included | <input type="checkbox"/> Diagram(s) Enhance Explanation |
| <input type="checkbox"/> Human Interest Appealing | <input type="checkbox"/> Overall Readability |
| <input type="checkbox"/> Clear Problem Statement | <input type="checkbox"/> Smooth Transition to Procedure |

GENERAL PROCEDURE

35 pts *Interpretation and explanation of mathematical concepts is clear, correct and easy to follow. The Procedure is implemented correctly. It is written in general terms and flows naturally into the Specific Example. Charts/diagrams enhance the paper. Correct function notation is used.*

0 pts *Interpretation and explanation of mathematical concepts is not explained in general terms or the explanation is so confusing it suggests significant misunderstanding. The reader is now screaming.*

- | | |
|---|---|
| <input type="checkbox"/> Procedure Overview Included | <input type="checkbox"/> Procedure is Easy to Follow |
| <input type="checkbox"/> Procedure Written in General Terms | <input type="checkbox"/> Results are Easily Identified |
| <input type="checkbox"/> Diagram(s) Enhance Explanation | <input type="checkbox"/> Clearly Marked Steps |
| <input type="checkbox"/> Appropriate / Clear Notation | <input type="checkbox"/> Level of Algebraic Detail is Appropriate |
| <input type="checkbox"/> Procedure Strategy is Valid | <input type="checkbox"/> Smooth Transition to Specific Example |

SPECIFIC EXAMPLE

10 pts *Processes/Strategies/Calculations used follow paper's General Procedure. They are easy to follow, accurate, complete and lead to a correct solution.*

0 pts *Processes/Strategies/Calculations used do not follow paper's General Procedure or are so unclear or contain substantial errors suggesting significant misunderstanding.*

- | | |
|---|---|
| <input type="checkbox"/> Example Coincides with Procedure | <input type="checkbox"/> Procedure is Correctly Implemented |
| <input type="checkbox"/> Clearly Marked Steps | <input type="checkbox"/> Level of Algebraic Detail is Appropriate |
| <input type="checkbox"/> Easy to Follow | <input type="checkbox"/> Solution Easily Identified |

CONCLUSION/SUMMARY

5 pts *Conclusion/Summary is easy to ready and is consistent with the Introduction.*

0 pts *Conclusion/Summary is missing, confusing or does not fit with the Introduction.*

Some things to keep in mind...

An "A" Report

The report clearly presents the problem and its solution. The report contains the appropriate sections: Introduction, Overview, Solution, Conclusion/Summary. The Introduction has a clear *statement of the problem* and contains some human interest. The Introduction flows logically and naturally into the Overview. The Overview contains a clear, well thought-out solution process. The report includes visual aids such as diagrams. Variables and constants are clearly identified. The process is described in general terms first before specific calculations are performed in the Example.

The report is reader friendly with the level of writing appropriate to the audience. It follows a common thread. There are very few if any errors in language. The report shows that the understanding of the topic goes beyond any specific examples used in class. The Summary/Conclusion provides a short recap and gives the reader clear direction for continuation.

A "C" Report

The report contains the appropriate sections but the sections are superficial. The report lacks evidence of logical reasoning, is disorganized or contains significant mathematical or language errors. All the pieces might be there and the report may even display evidence of significant effort but disorganization is rampant. The report shows evidence of minimal editing. The report is not reader-friendly; it lacks transitions or a common theme. The report reads like it was written in a hurry without much forethought or proof reading.

A "F" Report

The report does not even contain the appropriate sections. It lacks evidence of logical reasoning and it is severely disorganized. The sections are superficial and contain significant mathematical and language errors. Some of the pieces are there but the report displays evidence of minimal effort.

Introduction	Boring/Irrelevant	-	0	+	Boy do I want to read this!
Problem Statement	What is it?	-	0	+	Clear/Concise
Human Interest	Grass growing	-	0	+	Jurassic Park
Overview	Superficial	-	0	+	Clear/Concise Strategy Presented
Solution Procedure	All numeric	-	0	+	General Terms, Valid Approach, Correct
Example	Superficial	-	0	+	Thorough, Correct
Summary	Hodgepodge, Random thoughts	-	0	+	Recaps key points, Answer's what's next?
Conclusion	Grass growing, the sequel	-	0	+	T-Rex invades Bend

The project should include plenty of diagrams which you can hand-draw or create with computer graphics. I am also happy to e-mail you my graphics. If you plan to use a word processor, you are more than welcome to stop by my office and I'll give you a quick tutorial to help in typing mathematics.

Feel free to bring a draft(s) by for my critique. I am happy to help you make this a project to be proud of.

