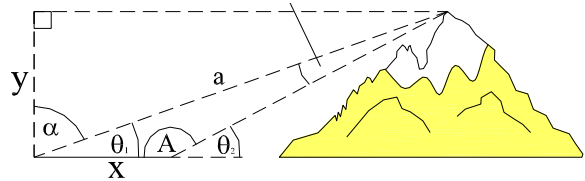


Perform your work **on separate paper** but write your answers on this page. Answers must be clearly **legible**. Where possible write answers as an **exact** value otherwise use **two** decimal accuracy. **Units** required. 20 pts

**Problem 1**

From sea level, two angular measurements ( $\theta_1$  and  $\theta_2$ ) are taken along the same bearing toward the top of a mountain. The distance between the angular measurements ( $x$ ) is also known. Find the mountain's height when  $\theta_1 = 27^\circ 12'$ ,  $\theta_2 = 32^\circ 48'$  and  $x = 1$  km.



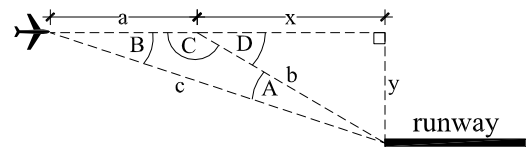
$H =$  \_\_\_\_\_

**Problem 2** Generalize problem 1 to find  $H$  as function of  $\theta_1$ ,  $\theta_2$ , and  $x$ .

$H(\theta_1, \theta_2, x) =$  \_\_\_\_\_

**Problem 3**

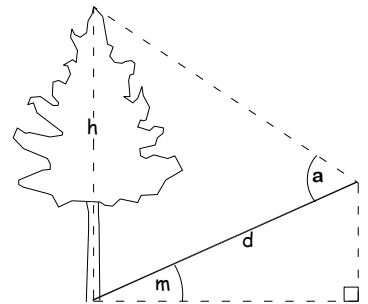
A plane is approaching an airport at 300 mph. The pilot takes an initial reading of  $12.7^\circ$  downward angle toward the beacon at the head of the runway and 12 minutes later, he takes a second reading of  $18.6^\circ$  downward angle. How far from the airport is the plane at the second reading? i.e. Find the vertical and horizontal distance.



$x =$  \_\_\_\_\_  $y =$  \_\_\_\_\_

**Problem 4**

165' uphill from the base of a tree the angle between the hill and a line to the top of a vertical tree is measured and found to be  $62^\circ$ . The hill is sloped at 15%. What is the height of the tree?



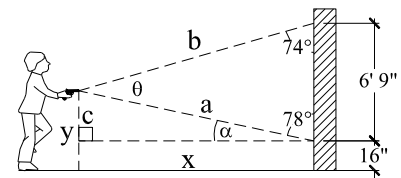
$h =$  \_\_\_\_\_

**Problem 5** Give the height as a function of  $m$ ,  $a$  and  $d$ .

$h(m, a, d) =$  \_\_\_\_\_

**Problem 6**

Forensic evidence shows that one bullet entered the wall 16" off the floor and at an angle of  $78^\circ$  off the vertical. A second bullet entered the wall much higher at an angle of  $74^\circ$  from the vertical. In fact, there is 6' 9" between the holes. Assuming the shooter did not move between shots, from where was the gun fired?



$x =$  \_\_\_\_\_  $y =$  \_\_\_\_\_

Our final problems involve finding distances and angles that are related to compass directions. This process is often referred to as *triangulation*. Before continuing, it might behoove you to review bearings, azimuth and their relation to the standard angle.

Although we previously made use of overlaying an  $(x, y)$ -coordinate system on geographic geometry, here we are limited to a single oblique triangle so the Law of Sines or Law of Cosines may be just as efficient. However, when multiple triangles are involved, the  $(x, y)$  system can be more efficient. Do these either way.

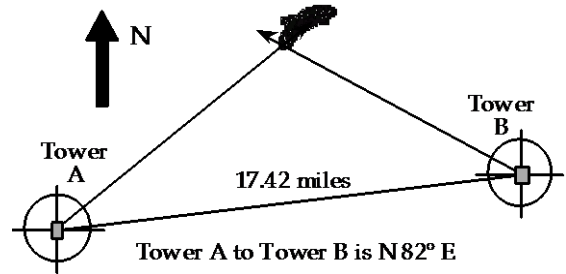
**Problem 7**

At 12:00 noon, a sailboat is heading N  $37^\circ$  W at 8 knots. At 2:15 pm the boat tacks to S  $44^\circ$  W. On that heading the sailboat makes 10 knots. Assuming the boat does not change course again, how far in a direct line (nautical miles) is the boat from its noon position at 3:30 pm?



**Problem 8**

Smoke is sighted by Lookout A @ N 53° E and soon thereafter by Lookout B @ N 58° W. Lookout B is situated 17.42 mi N 82° E of Lookout A. Find the distance from lookout A to the fire.

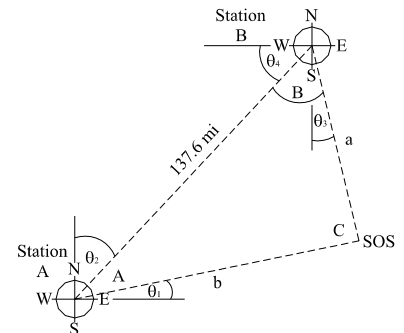


If Tower A is at (0, 0), what are the coordinates of the fire?

(x, y) = \_\_\_\_\_

**Problem 9**

A fishing boat sends out an SOS from somewhere offshore. USCG Station A picks up the signal on a heading of 78.4° azi. At the same time, USCG Station B picks up the SOS signal on a heading of 163.7° azi. The two stations are 137.6 miles apart with Station B located N 43.2° E of Station A. How far is the fishing boat from each station? What are the coordinates of the SOS if USCG Station A is the origin?

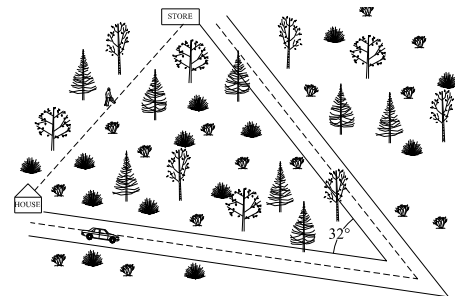


From A to SOS \_\_\_\_\_ From B to SOS \_\_\_\_\_

(x, y) \_\_\_\_\_

**Problem 10**

George wants to know if it's faster to drive or walk to the grocery store. If he drives, he must drive for 1.6 mi on 1<sup>st</sup> St, make a 32° turn and then drive 2.3 mi on 2<sup>nd</sup> Ave. If he walks, he can walk through the forest and take a straight line to the store. He can only drive at 20 mph but he can walk at 4 mph.



(a) How long will it take him to drive? \_\_\_\_\_

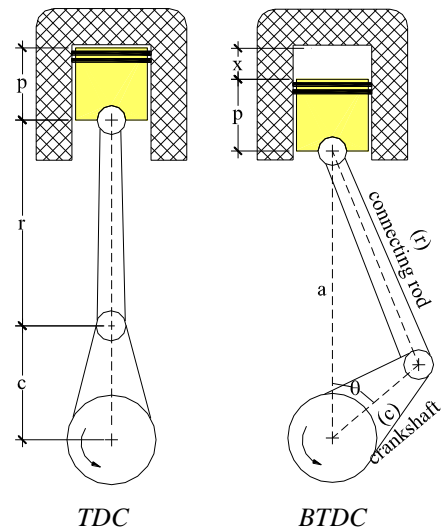
(b) How long will it take to walk? \_\_\_\_\_

**BONUS I**

In the 4-stroke engine, each ignition cycle has: (1) an upward stroke to compress and ignite the fuel mixture, (2) a downward power stroke, (3) an upward stroke to exhaust the burnt fuel and (4) a downward stroke to intake fresh fuel mixture.

When the piston is at its maximum height, the fuel mixture experiences maximum compression. This position is referred to as *Top Dead Center (TDC)*. Ignition actually occurs just prior to TDC referred to as *Before Top Dead Center (BTDC)*. Since we cannot easily see the internal position of the piston, its position is inferred from the rotational position of the crankshaft which is easy to see from the exterior.  $\theta = 0^\circ$  is generally aligned with TDC so  $180^\circ$  corresponds with the piston at the very bottom of the cylinder. Crucial positions, such as the position of the piston when the sparkplug fires (called ignition timing) is usually given in degrees BTDC. Our task is to convert crucial engine positions to crankshaft rotational position.

A particular 4-stroke engine is designed so that the sparkplug fires when the piston is 0.120" below maximum extension. The rod connecting the piston to the crankshaft is 8.315" and the distance from the center of the crankshaft to the connecting rod bearing is 3.265".



At what angle should the sparkplug be set to fire? \_\_\_\_\_

**BONUS II**

If the engine is turning at 6000 rpm, how much time elapses between the sparkplug firing and the piston reaching TDC?

t = \_\_\_\_\_