In the real world, it is not uncommon to take pieces of 2-D material and mold them into 3-D shapes. Clothes are a good example. Creating the finished product as a trial and error process is can be ineffective. With mathematics, we should be able to make the process much more efficient. Our task is to learn some mathematical procedures that will enable us to switch between the 2-D and 3-D version of an object using trigonometry.

Most of us could easily determine the dimensions of the flat piece that, when rolled, would make a straight circular duct of a given dimension. But could we determine the dimensions of the two flat pieces that, when rolled, make a 90 elbow? See fig 1.

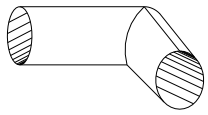


fig 1

Curved surfaces are obviously more complicated than polyhedrons and their intersections are more complicated yet. Working with triangles (right triangles no less) would be so much easier and that is where we begin.

The Problem

Suppose we have two slats of the same thickness. We want to find the miter angle (ϕ) and the bevel angle (β) (fig 2) so that the two pieces will make a symmetric 90° bend when joined as in a picture frame. See fig 3.

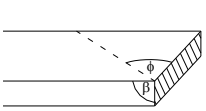


fig 2

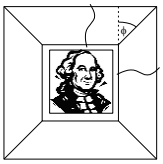


fig 3

Our natural intuition tells us that to make a flat picture frame we would miter each piece at 45° (ϕ = 45°) and cut the bevel angle plumb (90°). Joining these pieces would create the joint shown in fig 4.

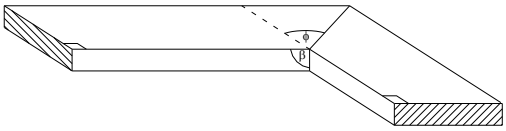


fig 4

But suppose we want to give more of a 3-D effect. See fig 5. If it is not already obvious, trial and error will quickly convince you that both the miter angle and the bevel angle must be adjusted. Our task is to determine the relationship between the rotation (θ) and the resulting cutting angles: the miter angle (ϕ) and the bevel angle (β).

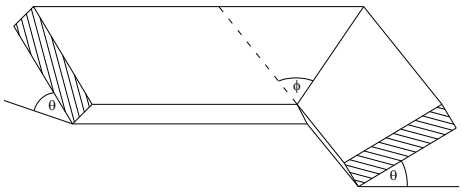


fig 5

Assignment

Solve the miter angle problem for the special case when $\theta = 45°$. That is, determine the bevel and miter angles. Write a report that details (in a neat organized fashion) the step by step process you used to find ϕ and β.

You may work together as a team and turn in one joint report or you may work individually. If you turn in a joint report you will normally receive the same grade but there are a few additional stipulations for joint reports.

In your write-up, I suggest you consider the outline here. See the reverse side for guidelines.

Title

Introduction

Step by Step Solution

Summary/Conclusion

One of the greatest difficulties here is the 3-D nature of the problem. Use the 3-D models provided. Begin with the special case $\theta = 45°$. That geometry is much simpler than with the general case. You'll need to use quite a bit of trigonometry and the Pythagorean Theorem. I suggest you work in an algebraic format rather than a numeric one. Once you have computed the angles use a protractor to measure the model to see if you are correct. Most likely, if you do make a math mistake the error will be obvious. I will tell you that only one of the two (either ϕ or β) is a 'nice' number. The other appears to be irrational.

Extra Credit

For 20 points extra credit, include the General Case as another section in your report. That is, determine the functional relations $\phi = f(\theta)$ and $\beta = g(\theta)$. You can use the following results to check your functions.

θ	ϕ	β
60º	26.565º	127.761º
70º	18.882º	131.641º

Guidelines

- Your paper must be typed or neatly handwritten
- Attach (staple) the grading rubric to your report
- All the pertinent information including diagrams must be in your report.
- There should be a **Title** and 3 distinct sections: **Introduction, Solution, and Conclusion**.
- Your **Introduction** should include some human interest to motivate the paper.
- Your **Introduction** should include a clear problem statement (your paper's purpose) *in your own words* so that someone not familiar with this assignment would understand the purpose of the paper.
- The **Solution** should contain a brief overview of your approach to solving the problem.
- The **Solution** must clearly show/describe the calculations in a step by step process.
- Your paper must contain at least one well placed diagram that is appropriately labeled and enhances your paper. If you use a variety of diagrams they should be well placed (not just grouped at the beginning or end of the paper).
- Your **Conclusion** should summarize the final results in a table of some kind.
- Your **Conclusion** should coincide with the theme you used in your **Introduction**.
- Team Projects must include a log of activities as well as a team assessment.

Some things to keep in mind...

An "A" Report
The report clearly presents the problem and its solution. The report contains the appropriate sections: *Introduction*, *Solution*, *Conclusion*. The Introduction has a clear *statement of the problem* and contains some human interest. The Introduction flows logically and naturally into the Solution. The Solution contains a clear, well thought-out solution process followed by a step by step explanation. The report includes visual aids such as diagrams. Variables and constants are clearly identified. The process is described in general terms first before specific calculations are performed in the Solution.

The report is reader friendly with the level of writing appropriate to the audience. It follows a common thread. There are very few if any errors in language. The report shows that the understanding of the topic goes beyond the specific examples used in class. The Conclusion provides a recap of results in a table and coincides with the Introduction.

A "C" Report
The report contains the appropriate sections but the sections are superficial. The report lacks evidence of logical reasoning, is disorganized or contains significant mathematical or language errors. All the pieces might be there and the report may even display evidence of significant effort but disorganization is rampant. The report shows evidence of minimal editing. The report is not reader-friendly; it lacks transitions or a common theme. The report reads like three people wrote it in a hurry without much coordination or proof reading.

An "F" Report
The report does not even contain the appropriate sections. It lacks evidence of logical reasoning and it is severely disorganized. The sections are superficial and contain significant mathematical and language errors. Some of the pieces are there but the report displays evidence of minimal effort.

Introduction	Boring/Irrelevant	–	0	+	Boy do I want to read this!
Problem Statement	What is it?	–	0	+	Clear/Concise
Human Interest	Grass growing	–	0	+	Jurassic Park
Overview	All numeric	–	0	+	General Terms, Algebraic Format
Solution Procedure	Superficial	–	0	+	Thorough
Conclusion	Hodgepodge, Random thoughts	–	0	+	Recaps key points, Answer's what's next?
Conclusion	Grass growing, the sequel	–	0	+	T-Rex invades Bend

Grading Rubric **NAME** _____

OVERALL FORMAT- Layout/Organization/Presentation

35 pts	*Typed with clearly readable font or neatly handwritten. Uses title and other clarifying headings. Layout and information organization/presentation flows for easy readability.*
0 pts	*Readability is significantly hindered by font type, handwriting, layout and/or information presentation.*

☐	Title, Use of Headings	☐	Appropriate / Clear Notation
☐	Section Breaks Apparent	☐	Appropriate Layout of Multiple Steps
☐	Appropriate use of White Space	☐	Spelling / Grammar
☐	Easy on the Eyes / Overall Readability	☐	Appropriate for Audience
☐	Clear Well Placed Diagram(s)	☐	First Impression

INTRODUCTION

10 pts	*Clear, easy to read. Provides motivation, includes clear problem statement and flows naturally into the General Procedure. Makes the reader want to continue reading. Diagrams enhance Introduction.*
0 pts	*Quite confusing / disorganized. Motivation missing or unclear. Problem statement missing, awkward or incorrect. Makes the reader want to tear out their hair (even if they are already bald).*

☐	Human Interest Included and Appealing	☐	Overall Readability
☐	Clear Problem Statement	☐	Smooth Transition to Solution

SOLUTION

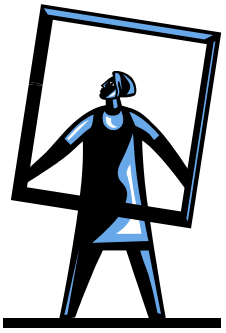
50 pts	*Processes/Strategies/Calculations used follow paper's General Procedure. They are easy to follow, accurate, complete and lead to a correct solution. Charts/diagrams enhance the paper.*
0 pts	*Processes/Strategies/Calculations used do not follow paper's General Procedure or are so unclear or contain substantial errors suggesting significant misunderstanding. The reader is now bald and screaming.*

☐	Solution Procedure Overview Included	☐	Diagram(s) Enhance Explanation
☐	Procedure is Valid	☐	Procedure is Easy to Follow
☐	Clearly Marked Specific Steps	☐	Appropriate Calculation Detail
☐	Result(s) Easily Identified	☐	Results Presented in a Table

CONCLUSION/SUMMARY

5 pts	*Conclusion/Summary has results presented in a Table, it is easy to ready and is consistent with the Introduction.*
0 pts	*Conclusion/Summary is missing, confusing or does not fit with the Introduction.*

The Saga of Joe's Olde Frame Shoppe



Introduction

Young Joe has had a life long dream of having his very own frame shop. Naturally he plans to provide the most elaborate frames: 3-D effects, gilded wood and elaborate mattes. All that he needs for perfect joints is the exact angles for his cuts. With his hand-dandy TI and a little trigonometry he is poised to tackle the problem….

The Problem

Take two slats of the same thickness. We want to find the miter angle (ϕ) and the bevel angle (β) (fig 1) ….

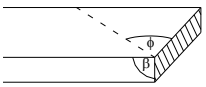


fig 1

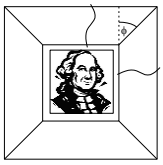


fig 2

Variables/Constants

θ is the angle the frame slat will be tilted off the wall (see fig 3).
ϕ is the miter angle, the angle cut across the face of the slat.
β is the bevel angle, the angle cut through the slat.
…

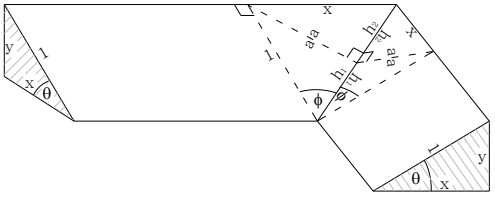


fig 3

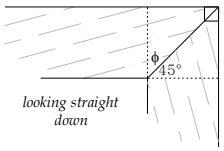
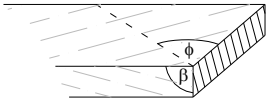
Solution Overview

We begin by making some 3-D models. See fig 3. Using the 3-D model for assistance …

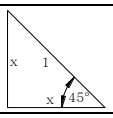
Solution

We are now ready to find the relation between θ and ϕ and β …

CASE θ = 0°

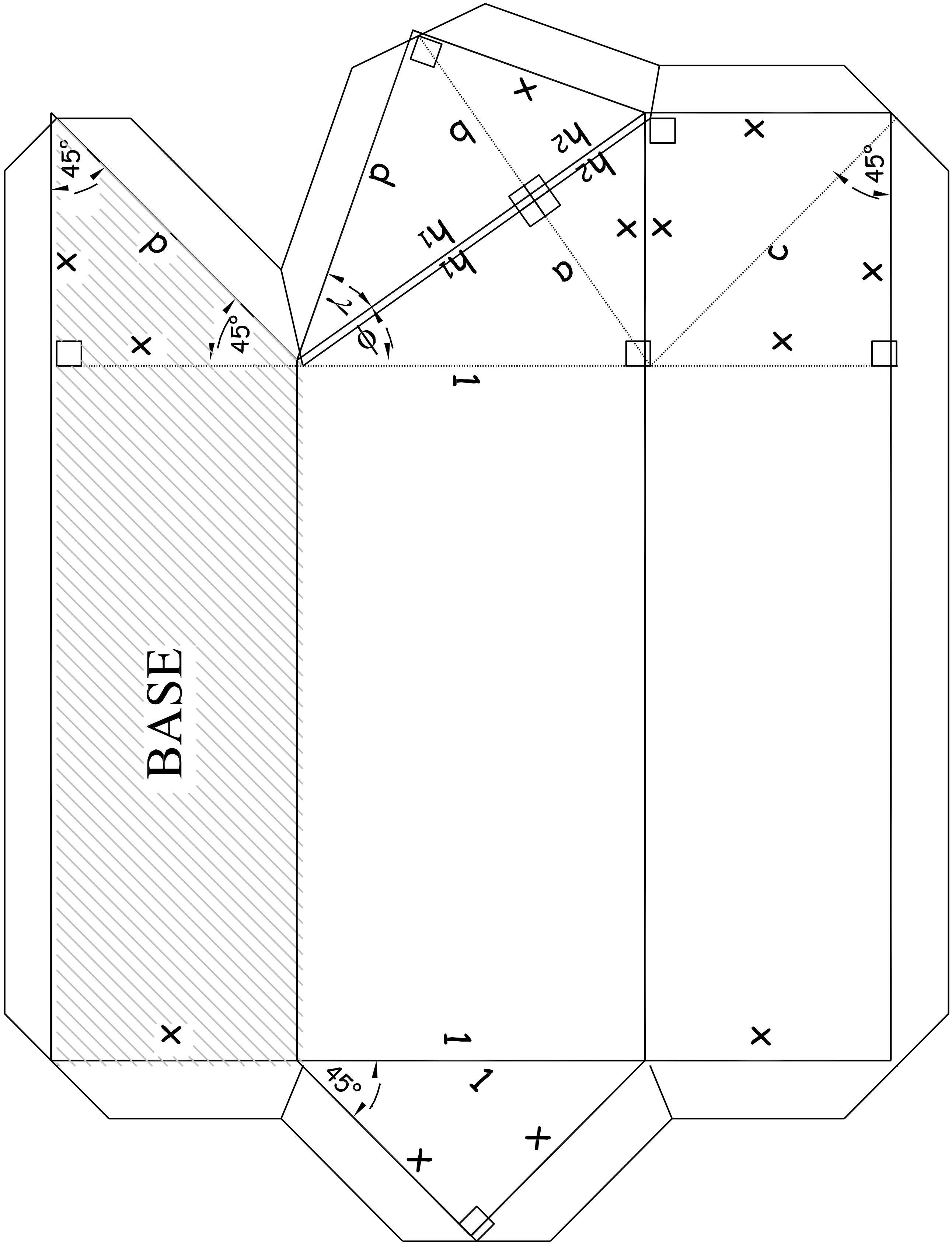
Algebra/Numeric Manipulation	Explanation & Diagrams
ϕ = 45°	*The projected angle in the horizontal plane (looking straight down) is always 90°. Since the slat is in the horizontal plane ϕ must split 90°. Half of 90° is 45°.* 
β = 90°	*Since the slat is horizontal, the bevel angle is perpendicular to the horizontal. Hence, the bevel angle is 90°.* 

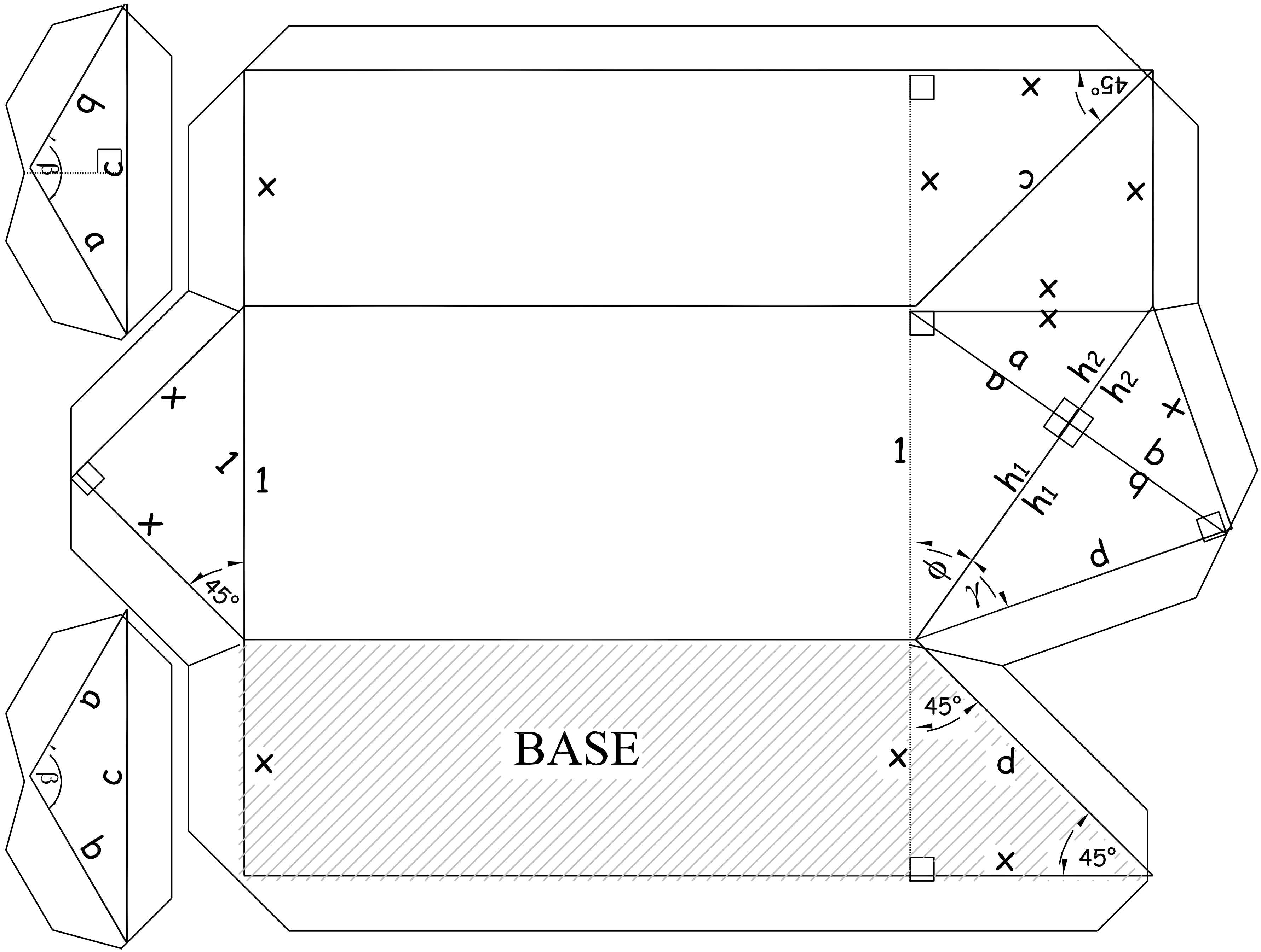
CASE θ = 45°

	Algebra/Numeric Manipulation	Explanation & Diagrams
Step 1	x = 1·cos θ = cos 45° = $\sqrt{½}$	*Side x is adjacent to angle θ so we have* *x = (hypotenuse)(cos θ)* 
Step 2	…	…

Summary/Conclusion

θ	ϕ	β
0°		
45°		
θ		


BASE
45°
45°
45°
45°
d
b
a
c
h_1
h_2
1
1
1
x



BASE
45°
45°
45°
45°
x
1
1
a
b
c
d
h1
h2
φ
γ
β
β