Mth 112 **Franz Helfenstein** **Name**

Working with (x,y)-points

As we have seen, all trigonometric ratios are closely tied to the unit circle. However, although not all (x, y)-points lie on the unit circle, every (x, y)-point can be thought of as lying on a circle of radius r and rotated by θ. As we see in Fig 1, whether θ is positive or negative, whether θ is less than or greater than 90°, each (r, θ) combination corresponds with a distinct (x, y)-point. In fact,

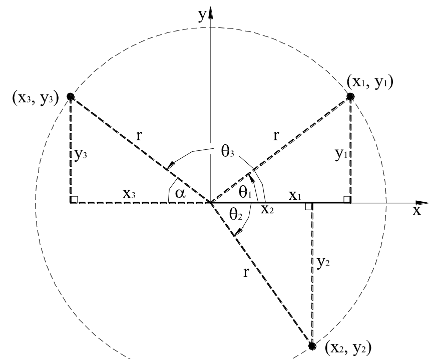


Fig 1 Trig Ratios on the General Circle

$$\cos(\theta) = \frac{x}{r} \quad \rightarrow \quad x = r\cos(\theta)$$

$$\sin(\theta) = \frac{y}{r} \quad \rightarrow \quad y = r\sin(\theta)$$

$$\tan(\theta) = \frac{y}{x} \quad \rightarrow \quad m = \frac{y}{x} = \tan(\theta)$$

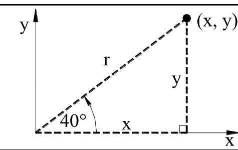
Example 1 Find the (x, y)-position for the polar coordinate (r, θ).

1) (r, θ) = (20, 40°)

$$x = r\cos(\theta) = 20\cos(40°) \approx 15.3$$

$$y = r\sin(\theta) = 20\sin(40°) \approx 12.9$$

$$\boxed{(x, y) \approx (15.3, 12.9)}$$



Traverses

A *traverse* is a sequence of connected straight-line segments on the surface of the Earth. Think of a traverse as a hike where you can only walk in a straight line along any segment of the hike.

In traverses we assume the distances are small enough so that the Earth's curvature is negligible. Thus, the geography can be overlaid with an (x, y)-coordinate system. Overlaying the traverse with an (x, y)-coordinate system is the key to simplifying a complex problem.

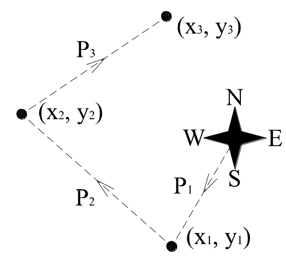
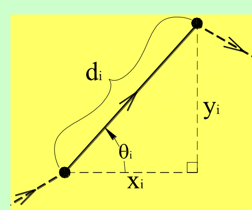


Fig 2 Typical Taverse

Typical questions for a traverse might be: (a) "Where are we now?", (b) "How far have we come?", (c) "How far and in what direction is the starting point?", (d) "How much area have we encircled?" etc.

Each segment of the traverse is referred to as a *leg* of the traverse. A traverse may be *opened* or *closed*. One of the most common computations for a traverse is to check the computation that should close the traverse against the actual data of that leg. The error between the two is a direct result of the accuracy used in making the traverse's measurements.

Traverse Procedure



1) Convert all directions to their equivalent θ-angles.
2) For each leg of the traverse, compute:

$$x_i = d_i\cos(\theta_i) \text{ and } y_i = d_i\sin(\theta_i).$$

3) The (x, y)-position at any node is given by:

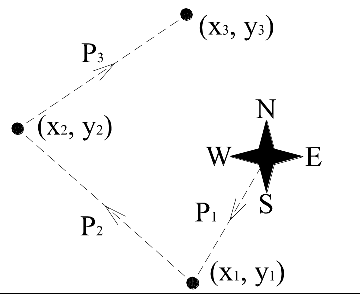
$$(x, y) = (\Sigma x_i, \Sigma y_i) \qquad \Sigma \text{ mean sum up all the } x_i\text{'s}$$

4) From any (x, y)-position the distance back to origin can be found using: $d = \sqrt{x^2 + y^2}$.
5) The direction from the origin to (x, y) is found using: $\theta = \tan^{-1}(y/x)$.
6) The direction from (x, y) to the origin is found using: $\theta = \tan^{-1}(-y/-x)$.

For (x, y) points in the 2nd quadrant, the inverse tangent will need to be adjusted by adding 180° to θ to get the true direction. For (x, y) points in the 3rd quadrant, the inverse tangent will need to be adjusted by subtracting 180° from θ to get the true direction.

Example 2

A grizzly bear wanders 800 m along a river that flows S 20° W, then travels 1,400 m due NW and finally ambles 1,500 m N 36°E. Where is the bear relative to its starting point?



1) $r_1 = 800$ m, $\theta_1 = -90° + -20° = -110°$

$r_2 = 1{,}400$ m, $\theta_2 = 90° + 45° = 135°$

$r_3 = 1500$ m, $\theta_3 = 90° - 36° = 54°$

2) $x_1 = 800$ m cos (-110°) | $y_1 = 800$ m sin (-110°)

$x_2 = 1{,}400$ m cos (135°) | $y_2 = 1{,}400$ m sin (135°)

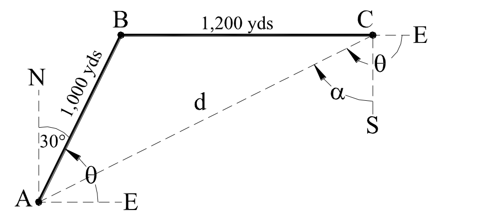
\+ $x_3 = 1{,}500$ m cos (54°) | + $y_3 = 1{,}500$ m sin (54°)

$x \approx$ -382 m | $y \approx$ 1,452 m

Thus, the bear is 382 m west and 1,452 m north of its starting position.

Problems

1) A tank travels 1,000 yds from pt A to pt B on a N 30° E bearing then changes direction and travels due east 1,200 yds from pt B to pt C. What direction and how far must the tank travel to return to its starting point most directly?



Leg	Bearing	θ	d_i	x_i	y_i
1					
2					
Net					

Direction Home (bearing) ______________________

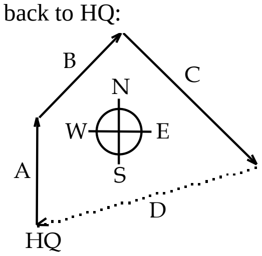
Distance Home (mi) ________________________

2) Find direction and distance back to HQ:

A:2 mi N

B: 2 mi NE

C: 3 mi SE



Direction Home (bearing) ___________

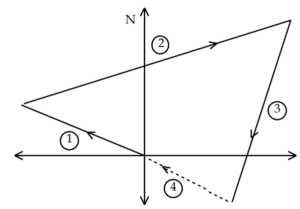
Distance Home (mi) ________________

3) Find the distance and heading for Path 4.

Path$_1$: 4,412 ft @ N 67° W

Path$_2$: 7,305 ft @ N 72° E

Path3: 4,892 ft @ S 20° W



Direction Home (bearing) ___________________

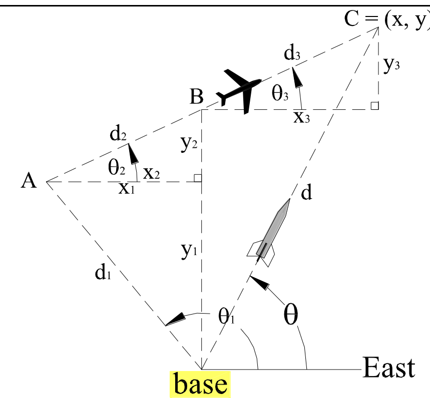
Distance Home (mi) ________________________

4) Although this example is not exactly a traverse, it can benefit from the same procedure.

A jet flying at 560 mph and heading N 65° E passes Pt A which is N 40° W of the missile base. 5 minutes later, the jet is due north of the base. At that moment, a missile is fired at the jet with a 10 min flight time.

What should be the missile's heading? ___________________________

How fast must it fly to intersect the jet in exactly 10 min? ____________



5) At 12:00 noon, a sailboat is heading N 37° W at 8 knots. At 1:15 pm the boat tacks to S 44° W. On that heading the boat makes 10 knots. Assuming the boat does not change course again,

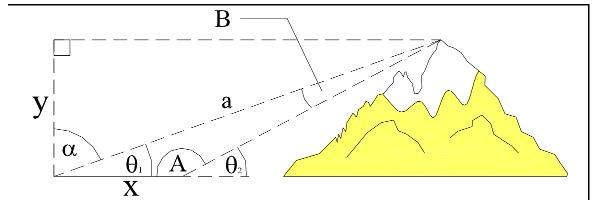
How far (knots) in a direct line is the boat from its noon position at 2:00 pm? _____________

Answers

1) 3.76 mi, S 70° W	2) 1,360 ft, 153° N 63° W area = 13,875,517	3) 1360' N 63.1° W

Example 6

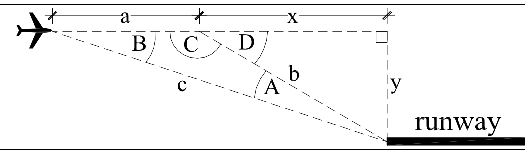
From sea level, two angular measurements (θ_1 and θ_2) are taken along the same bearing toward the top of a mountain. The distance between the angular measurements (x) is also known. Find the mountain's height when $\theta_1 = 27° \; 12'$, $\theta_2 = 32° \; 48'$ and x = 1 km.



Example 7 Generalize the problem in Example 6

Example 8

A plane is approaching an airport at 300 mph. The pilot takes an initial reading of 12.7° downward angle toward the beacon at the head of the runway and 12 minutes later, he takes a second reading of 18.6° downward angle. How far from the airport is the plane at the second reading? i.e. Find the vertical and horizontal distance.

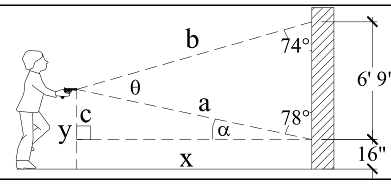


Example 9

165' uphill from the base of a tree the angle between the hill and a line to the top of a vertical tree is measured and found to be 62°. The hill is sloped at 15%. What is the height of the tree?

Example 10

Forensic evidence shows that one bullet entered the wall 16" off the floor and at an angle of 78° off the vertical. A second bullet entered the wall much higher at an angle of 74° from the vertical. In fact, there is 6' 9" between the holes. Assuming the shooter did not move between shots, from where was the gun fired?

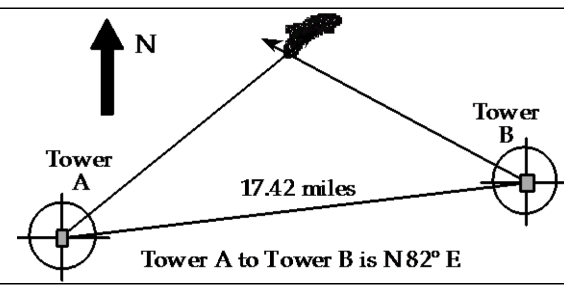


Our final examples involve finding distances and angles that are related to compass directions. This process is often referred to as *triangulation*. Before continuing, it might behoove you to review bearings, azimuth and their relation to the standard angle.

Although we previously made extensive use of overlaying an (x, y)-coordinate system on geographic geometry, here we are limited to a single oblique triangle so the Law of Sines is just as efficient. However, when multiple triangles are involved, the (x, y) system can be more efficient.

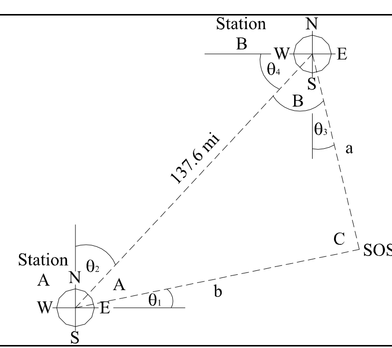
Example 11

Smoke is sighted by Lookout A @ N 53° E and soon thereafter by Lookout B @ N 58° W. Lookout B is situated 17.42 mi N 82° E of Lookout A. Find the distance from each lookout to the fire.



Example 12

A fishing boat sends out an emergency SOS from somewhere offshore. CG Station A picks up the signal on a heading of 78.4° azi. At the same time, CG Station B picks up the SOS signal on a heading of 163.7° azi. The two stations are 137.6 miles apart with Station B located N 43.2° E of Station A. How far is the fishing boat from each station?



Example 13

At 12:00 noon, a sailboat is heading N 37° W at 8 knots. At 1:15 pm the boat tacks to S 44° W. On that heading the boat makes 10 knots. Assuming the boat does not change course again, how far (direct line) is the boat from its noon position at 2:00 pm?

Typically, when a chimney or other rectangular structure protrudes through a roof, the uphill side of the protruding structure allows snow and debris to collect in 'dead' spots. This causes obvious problems.

To eliminate such 'dead spots', triangular surfaces are built that divert water downhill.

In roof construction, a *cricket* refers to the structure (usually a combination of pyramids and/or tetrahedrons) that acts as a water diverter. Crickets are built on pitched roofs as well as flat roofs.

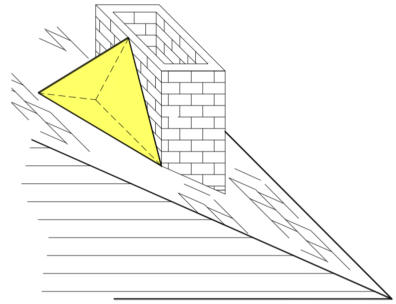


Fig 1. Typical Chimney Cricket

Example 6

A 42" wide chimney protrudes from a 6/12 pitched roof. A cricket is to be built that spans the width of the chimney, extends vertically up the chimney 30" and up the roof 36". What are the dimensions of each half of the cricket? That is, what size & shape triangles must be cut to form the cricket?

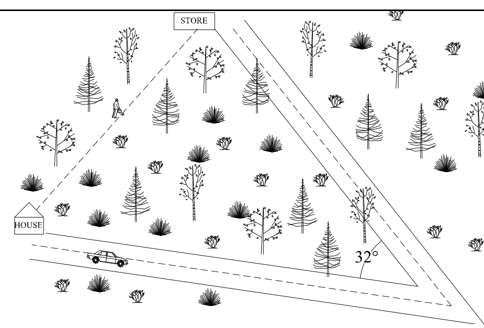
Example 7

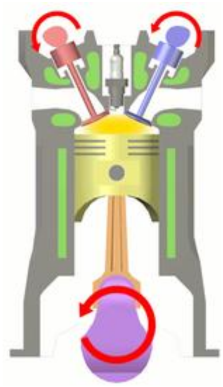
George wants to know if it's faster to drive or walk to the grocery store. If he drives, he must drive for 1.6 mi on 1st St, make a 32° turn and then drive 2.3 mi on 2nd Ave. If he walks, he can walk through the forest and take a straight line to the store. He can only drive at 20 mph but he can walk at 4 mph.

(a) How long will it take him to drive?

(b) How long will it take to walk?

(c) If George could ride his bike through the forest, how fast would he need to ride to be as quick as going by car?

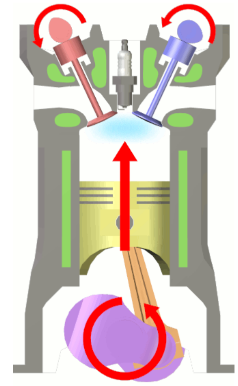




TDC

Internal combustion engines come in a variety of forms. The most common is the 4-stroke gasoline engine. In the 4-stroke engine, each ignition cycle has: (1) an upward stroke to compress and ignite the fuel mixture, (2) a downward power stroke, (3) an upward stroke to exhaust the burnt fuel and (4) a downward stroke to intake fresh fuel mixture.

When the piston is at its maximum height, the fuel mixture experiences maximum compression. This position is referred to as *Top Dead Center* (TDC).



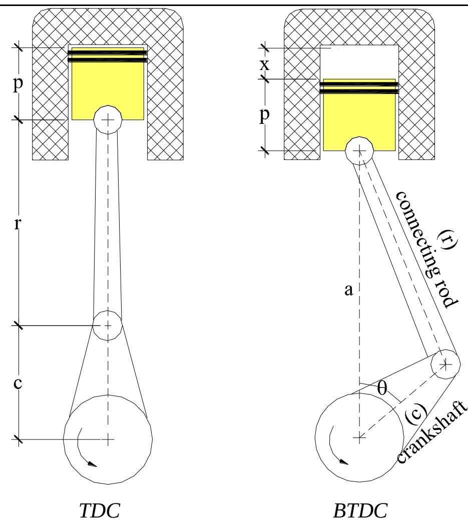
120° BTDC

Since we cannot easily see the internal position of the piston, its position is inferred from the rotational position of the crankshaft which is easy to see from the exterior. θ = 0° is generally aligned with TDC so 180° corresponds with the piston at the very bottom of the cylinder. Ignition actually occurs just prior to TDC referred to as *Before Top Dead Center* (BTDC). Crucial positions, such as the position of the piston when the sparkplug fires (called ignition timing) is usually given in degrees BTDC.
Our task is to convert crucial engine positions to crankshaft rotational position.

Example 8

A particular 4-stroke engine is designed so that the sparkplug fires when the piston is 0.120" below maximum extension. The rod connecting the piston to the crankshaft is 8.315" and the distance from the center of the crankshaft to the connecting rod bearing is 3.265".

At what angle should the sparkplug be set to fire?



Example 9

A scissor truss is to be constructed from 2 x 6 lumber with the roof rafter at a 7/12 pitch and the ceiling rafter at a 3/12 pitch. The entire span is 28'. See diagram. Find the length of support member c. Note: 2 x 6 lumber is 5 ½" wide