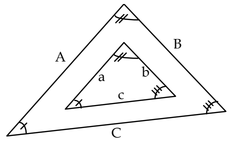
Similar Triangles

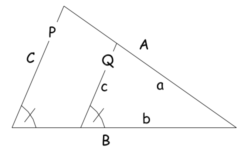


corresponding angles match

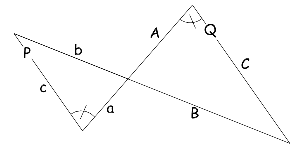
Compare Like Ratios to Like Ratios

Examples:

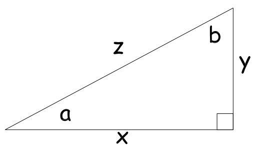
$\frac{\text{Large}}{\text{Small}} \quad \frac{A}{a} = \frac{B}{b}$ or $\frac{\text{Left}}{\text{Right}} \quad \frac{A}{B} = \frac{a}{b}$ etc.



imbedded triangles are similar when P || Q



reflected triangles are similar when P || Q



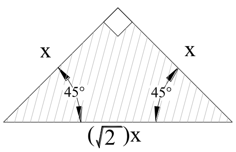
Slope
Grade
Pitch
Gradient
Tangent

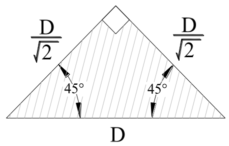
$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{y}{x}$ $\quad x^2 + y^2 = z^2$ $\quad a + b = 90°$

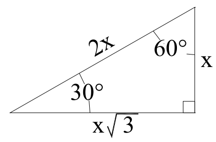
$x = \frac{z}{\sqrt{1 + m^2}}$ $\quad y = \frac{z\,m}{\sqrt{1 + m^2}}$

45°-45°-90° Triangles

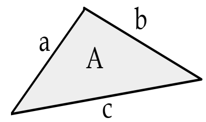




30°-60°-90° Triangles

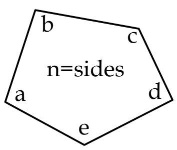


Heron's Formula



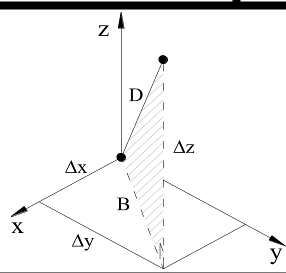
$s = \frac{a + b + c}{2}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$



total degrees = $(n-2)\,180°$

3D Pythagorean Theorem

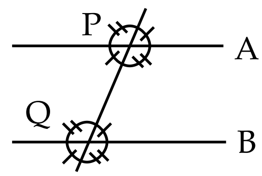


$\Delta x = x_2 - x_1$

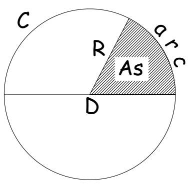
$\Delta y = y_2 - y_1$

$\Delta z = z_2 - z_1$

$D^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$



When A || B then <P = <Q



$A = \pi R^2$

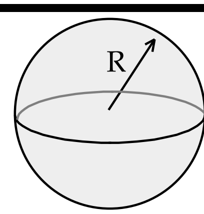
$D = 2R$

$C = \pi D$

$C = 2\pi R$

$A_S = \frac{\theta}{360°}\,\pi R^2$

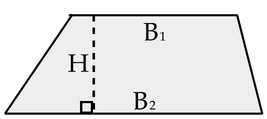
$\text{arc} = \frac{\theta}{360°}\,2\pi R$



$V = \frac{4\pi R^3}{3}$

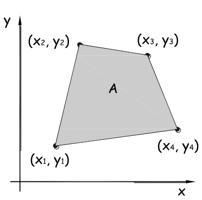
Surface Area = $4\pi R^2$

Trapezoids



$A = \frac{(B_1 + B_2)}{2}\,H$

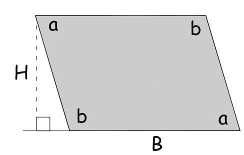
Polygon Area



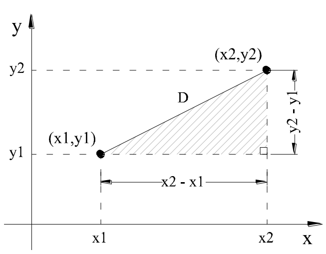
		$x_1\ y_1$		
	$x_2\ y_1$	$x_2\ y_2$	$x_1\ y_2$	
	$x_3\ y_2$	$x_3\ y_3$	$x_2\ y_3$	
	$x_4\ y_3$	$x_4\ y_4$	$x_3\ y_4$	
+	$x_1\ y_4$	$x_1\ y_1$	$x_4\ y_5$	+

$A = \left| \frac{[\quad] - [\quad]}{2} \right|$

Parallelograms



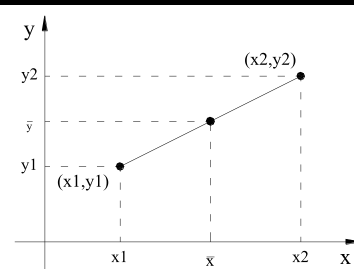
$A = B\,H$ $\quad a + b = 180°$



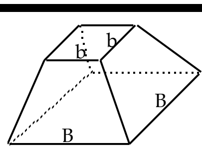
$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

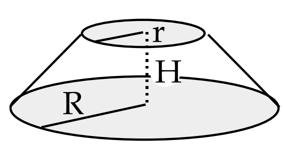
$D = \sqrt{\Delta x^2 + \Delta y^2}$



Volume of Frustums

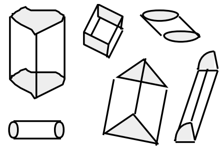


$V = \frac{(B^2 + B\,b + b^2)\,H}{3}$



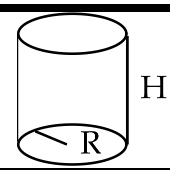
$V = \frac{\pi H}{3}\,(r^2 + Rr + R^2)$

Solids with *common cross-sections* (such as a barrel) all have similar volumes



$V = (A_{Base})H$

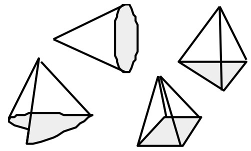
height is perpendicular to the base



$V = \pi R^2 H$

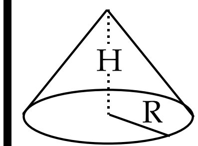
Surface Area = $2\pi RH + 2\pi R^2$

Solids with *similar cross-sections* (such as a cone) all have similar volumes



$V = \dfrac{(A_{Base})H}{3}$

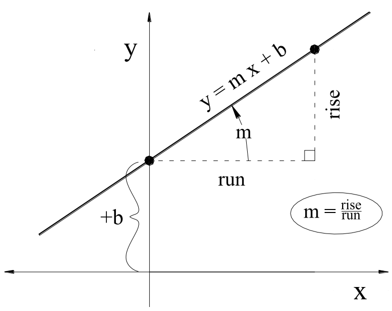
height is perpendicular to the base



Cone

$V = \dfrac{\pi R^2 H}{3}$

Lines



<u>To Graph y= mx + b</u>

① Plot (0, b) the y-intercept
② Plot a second point using m by shifting horizontally by run and shifting vertically by rise.
③ Draw graph

<u>To Graph ax + by = c</u>

Plot the intercepts
① set x = 0, solve for y and plot (0, y)
① set y = 0, solve for x and plot (x, 0
③ Draw graph

<u>To Find a Line's Equation</u>

① Find m: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

② Find b: b = y-intercept (i.e. x = 0)

if the y-intercept is between integers or off the grid b should be computed using $b = y_0 - m\,x_0$ where (x_0, y_0) is any **known** point

③ Once m & b are found: $y = m\,x + b$

Prefix	**Symbol**	**Factor**	**Example**
Giga	G	10^9	5 GigaBytes
Mega	M	10^6	6 Megahertz
Kilo	k	10^3	2 kilometers
Hecto	h	10^2	3 hectograms
Deka	da	10^1	5 dekaliters
Deci	d	10^{-1}	3 decigrams
Centi	c	10^{-2}	2 centimeters
Milli	m	10^{-3}	5 milligrams
Micro	µ	10^{-6}	4 micrograms
Nano	n	10^{-9}	5 nanoseconds

<u>Solving Linear Equations</u>

1) Remove Fractions: mult through by LCD
cancel fractions
2) Remove ( ): distributive rule; beware of negatives!
3) Combine Like Terms
4) Shift variable terms to one side (add/sub)
5) Shift all other terms to other side
6) Write as (coefficient) × (variable)
7) Divide out the Coefficient
8) Check the Answer

<u>Exponents</u>

$a^m a^n = a^{m+n}$

$\dfrac{a^m}{a^n} = a^{m-n}$

$a^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$

$(a\,b)^n = (a^n)\,(b^n)$

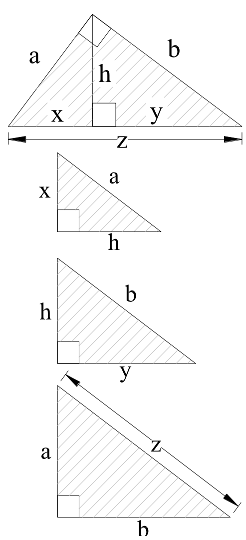
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$\sqrt{a} = a^{1/2} = a^{0.5}$

$\sqrt{ab} = \sqrt{a}\,\sqrt{b}$

$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

<u>Mitered Triangles</u>



<u>Quadratic Formula</u>

$ax^2 + bx + c = 0; \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

<u>Distributive Rule</u>

$\mathbf{a(b \pm c) = ab \pm ac} \qquad \mathbf{(b \pm c)a = ba \pm ca}$

$\dfrac{\mathbf{b \pm c}}{\mathbf{a}} = \dfrac{\mathbf{b}}{\mathbf{a}} \pm \dfrac{\mathbf{c}}{\mathbf{a}}$

<u>Factoring</u>

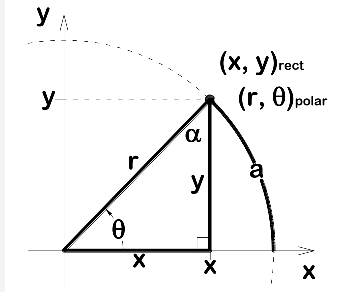
$\mathbf{ax + bx = (a + b)x}$

$\mathbf{ax + bx - cx = (a + b - c)x}$

Summary of Trig Relationships

Non-trig Relationships using Slope, m

$m = \frac{y}{x}$ $\qquad x^2 + y^2 = r^2$ $\qquad x = \frac{r}{\sqrt{1 + m^2}}$ $\qquad y = \frac{r\,m}{\sqrt{1 + m^2}}$



Standard Diagram

Arc Length and Velocity

$a = r\,\theta$ (θ in radians) $\qquad a = \frac{\theta°}{360°}\,2\pi\,r$ (θ in degrees) $\qquad 2\pi_{rad} = 360°$

$v = r\,\omega$ (θ in radians) e.g. (ft/sec = ft × rad/sec)

Trig Relationships

Given a right triangle when θ is known use these trig relationships to find a missing side

$\sin\theta = \frac{y}{r} = \frac{opp}{hyp}$

$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$

$\tan\theta = \frac{y}{x} = \frac{opp}{adj}$

Given a right triangle when θ is not known use these trig relationships to find an angle

$\theta = \sin^{-1}\left(\frac{y}{r}\right) = \sin^{-1}\left(\frac{opp}{hyp}\right)$

$\theta = \cos^{-1}\left(\frac{x}{r}\right) = \cos^{-1}\left(\frac{adj}{hyp}\right)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{opp}{adj}\right)$

x = r cos θ $\qquad y = r\sin\theta$ $\qquad \theta + \alpha = 90° = \pi/2_{rad}$ $\qquad 45° = \pi/4$ $\qquad 180° = \pi$ $\qquad 60° = \pi/6$ $\qquad 30° = \pi/12$

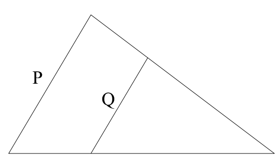
$\tan\theta = m$ $\qquad \theta = \tan^{-1} m$ $\qquad \sin\theta = \cos\alpha$ $\qquad \cos\theta = \sin\alpha$ $\qquad \sin^2\theta + \cos^2\theta = 1$ $\qquad \frac{\sin\theta}{\cos\theta} = \tan\theta$

Some Examples that do NOT require Trigonometry

Not every triangle requires trigonometry.

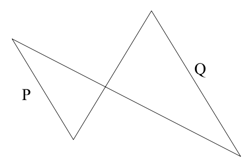
Don't forget to use similar triangles where possible.

Don't forget there are 180° in every triangle.



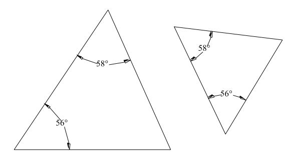
Imbedded Triangles, P∥Q

Use Similar Triangles



Reflected Triangles, P∥Q

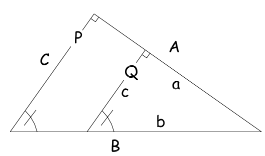
Use Similar Triangles


Two Angles Match

Use 180° in triangles
Use Similar Triangles

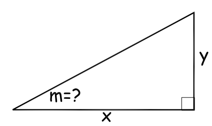
Only right triangles can use the Pythagorean Thm.

Slope only makes sense with a vertical vs. horizontal relationship.



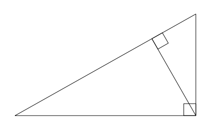
Imbedded Right Triangles

Use Similar Triangles
Use Pythagorean Thm.



Both legs known, slope missing

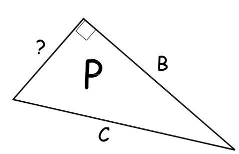
Use m = $\frac{rise}{run}$

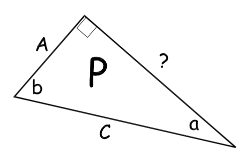

Mitered Right Triangles

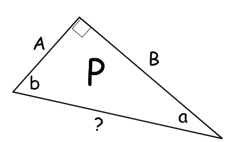
Use Similar Triangles
Use Pythagorean Thm.

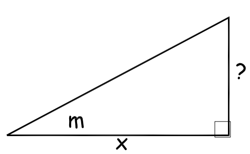
A right triangle with 2 sides known, one side missing

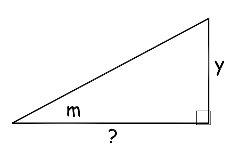
Use Pythagorean Thm.





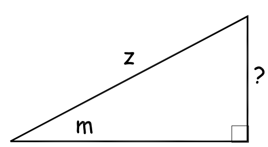


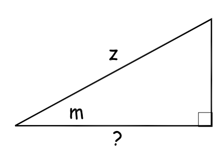




Slope and one leg known, other leg missing

Use m = $\frac{rise}{run}$



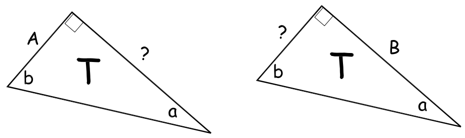


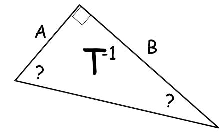
Slope and diagonal known, legs missing

Use $\quad x = \frac{z}{\sqrt{1 + m^2}}\ ;\quad y = \frac{z\,m}{\sqrt{1 + m^2}}$

Examples that do require Trigonometry

The tangent is used when





There is a right triangle

one angle and one leg known, other leg missing

Use $\tan\theta = opp/adj$

Both legs known, angles missing

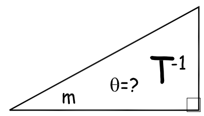
Use $\theta = \tan^{-1}(opp/adj)$

The parts of interest are:

an angle,

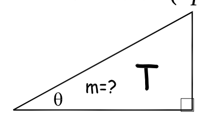
both legs,

the slope



Slope known, Standard angle missing

Use $\theta = \tan^{-1} m$



Standard angle known, slope missing

Use $\tan\theta = m$

The sine is used when

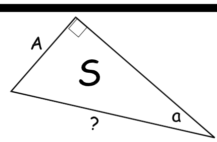
There is a right triangle

The parts of interest are:

an angle,

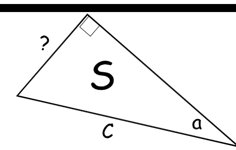
its opposite leg,

the diagonal



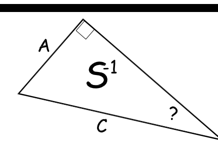
Angle, opposite leg known, hypotenuse missing

Use $\sin\theta = opp/hyp$



Angle, hypotenuse known, leg opposite angle missing

Use $\sin\theta = opp/hyp$



Hypotenuse, leg known; angle opposite leg missing

Use $\theta = \sin^{-1}(opp/hyp)$

The cosine is used when

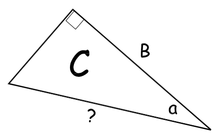
There is a right triangle

The parts of interest are:

an angle,

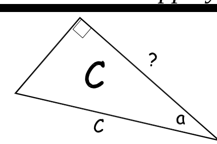
its adjacent leg,

the diagonal



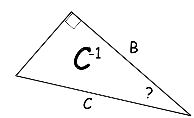
Angle, adjacent leg known, hypotenuse missing

Use $\cos\theta = adj/hyp$



Angle, hypotenuse known, adjacent leg missing

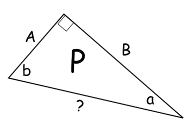
Use $\cos\theta = adj/hyp$



Known sides bracketing a missing angle

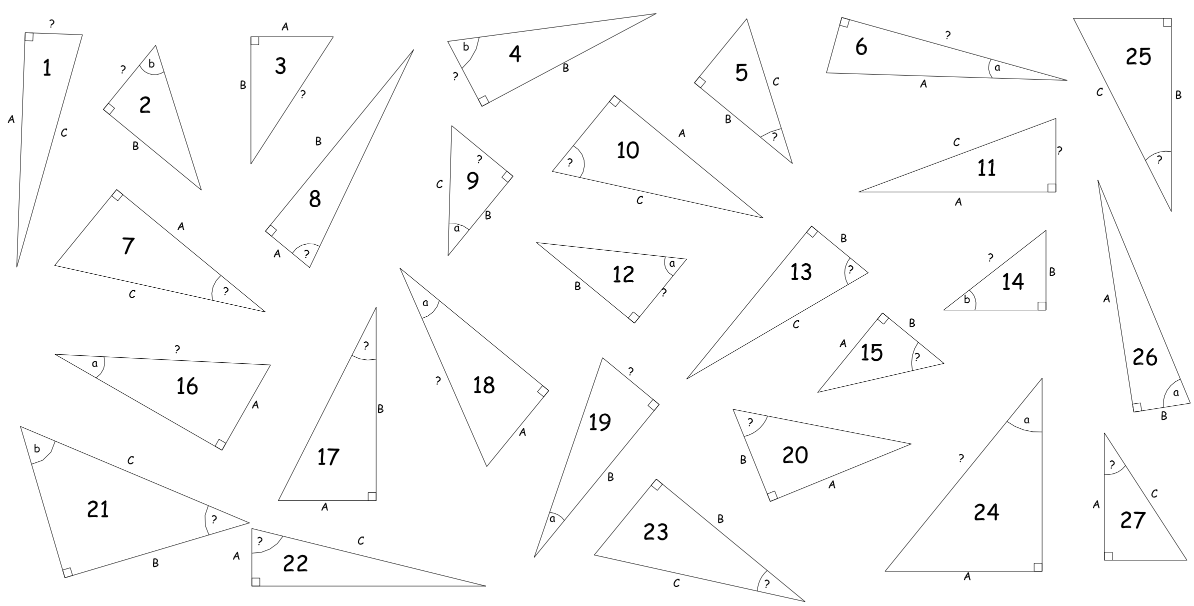
Use $\theta = \cos^{-1}(adj/hyp)$

When extra information is known there are multiple possibilities.

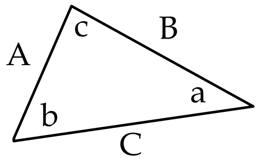
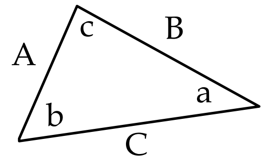


A right triangle with all sides known but angles missing

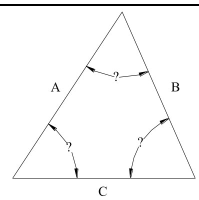
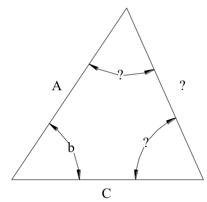
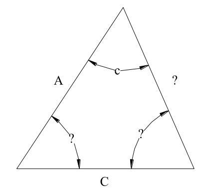
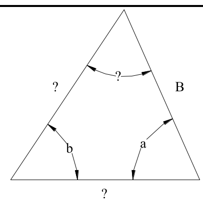
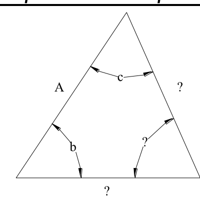
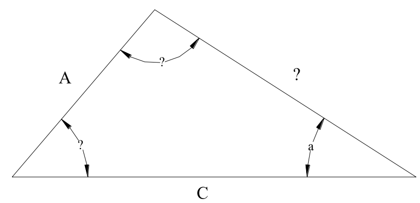
Use any inverse trig function



Non-Right Triangles use Law of Sines or Law of Cosines

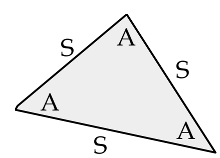
Use Law of Sines with	*Use Law of Cosines with*
2 angles + 2 sides with one part missing. *Often must first use sum of angles is 180°.*  **Law of Sines** $\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$	*3 sides known and missing angles or side-angle-side combo known with side opposite angle missing*  **Law of Cosines** $A^2 + B^2 - 2AB\cos\theta = C^2$ $\cos\theta = \frac{A^2 + B^2 - C^2}{2AB}$

Examples

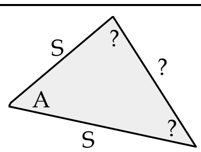
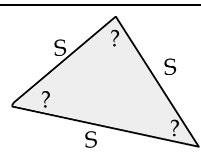
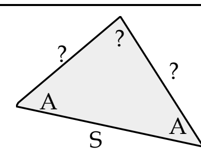
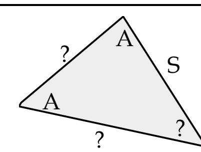
 3 Sides known, angles Missing *Use Law of Cosines to find first angle* *Many choices after that*	 side-angle-side combo known *Use Law of Cosines to find third side*	 2 Side-Opposite Angle Combos with 3 parts known *Use Law of Sines to find missing part* *Many choices after that*
 angle-angle-side combo known *Use 180° in triangle for 3rd angle* *Use Law of Sines to find sides*	 angle-side-angle combo known *Use 180° in triangle for 3rd angle* *Use Law of Sines to find sides*	 Be careful when using Law of Sines with Obtuse Triangles!

Summary of Basic Trigonometry:

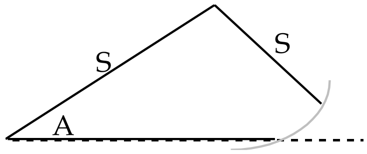
Using Trigonometry in a geometry problem involves triangles. The triangle(s) may be obvious but sometimes they are imbedded deceptively within a more complicated geometry. Generally, the task at hand is to use partial information given about the various triangles to determine all three angles (A) and all three sides (S) of each triangle.



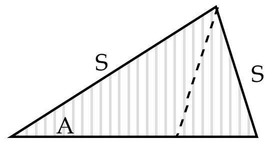
Given only SSS, SAS, ASA, AAS we always have a **unique triangle**. Thus, the remaining three parts of the triangle are **predetermined**. In each of the examples below the remaining three parts (?) can be found using trigonometry formulas.

 SAS	 SSS	 ASA	 AAS

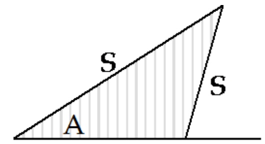
However, if given SSA two possible triangles can occur but only two. Trigonometry can still give us the unknown parts exactly if we know which of the two cases we have, an acute triangle or an obtuse triangle.



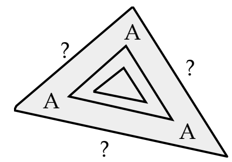
2 possibilities



acute triangle



obtuse triangle



If we have just the three angles known there are an infinite number of **similar triangles.** Thus, we must always have at least one side given to exactly determine all parts.

With the exception of AAA (similar triangles) knowing three of the six triangle parts allows us to find the remaining three parts (assuming we have a sketch for the SSA case).

In the case of right triangles, one angle (the right angle) is already known so we only need two additional parts to exactly determine all six parts. Right triangles are usually preferable because the relationships are simpler to work with.

Not every triangle requires trigonometry to determine its components. Often a triangle is oriented so that "slope" makes sense for one "angle" of the triangle. If a slope is given (or desired) we can often avoid trigonometry and still determine all the remaining parts of the triangle.

In Trigonometry Applications:

1) Find triangles inherent and useful to the problem. Label them.
2) Decide if a triangle is a right-triangle or non-right-triangle.
3) Determine which relation is required.
4) Use the relationship to set up an equation.
5) Solve the equation
6) Repeat as necessary

Some hints:

Only **right triangles** can use the **Pythagorean Theorem**.

Recall there are **180° in a triangle** and complementary angles sum to 90°.

Use **Sin, Cos or Tan** with **right triangles** when the **angle is known**.

Use **Sin^{-1}, Cos^{-1} or Tan^{-1}** with **right triangles** when the **angle is the unknown**.

When two sides form a "**t**" use **Tan or Tan^{-1}**.

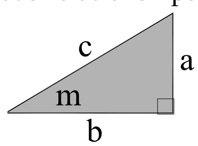
Right-triangles which involve the **hypotenuse and an angle** use either **Sin or Cos**.

Right-triangles which involve **two sides bracketing an angle** use **Cos**.

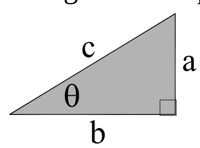
Right-triangles which involve the **hypotenuse** and a **side opposite an angle** use **Sin**.

Non-right triangles use **Law of Sines** or **Law of Cosines**.

When a right triangle is in its standard position the various relationships are easier to recognize and apply.



standard position with slope



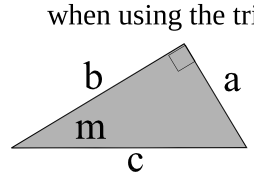
standard position with angle

$m = \frac{a}{b}$; $a^2 + b^2 = c^2$; $b = \frac{c}{\sqrt{1 + m^2}}$; $a = \frac{c\,m}{\sqrt{1 + m^2}}$

$\tan\theta = \frac{rise}{run}$; $\sin\theta = \frac{rise}{hyp}$; $\cos\theta = \frac{run}{hyp}$

$a = c\sin\theta$ $b = c\cos\theta$

When a right triangle is in a random position be careful when using the trigonometric relationships.

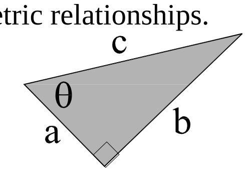


be careful using slope here

$m = \tan\theta = \frac{a}{b}$

$\theta = \tan^{-1}\frac{opp}{adj} = \tan^{-1}\frac{a}{b}$

$\theta = \sin^{-1}\frac{opp}{hyp}$; $\theta = \cos^{-1}\frac{adj}{hyp}$



slope is inapplicable here

$\tan\theta = \frac{opp}{adj} = \frac{b}{a}$

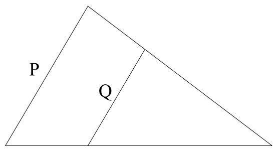
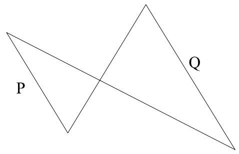
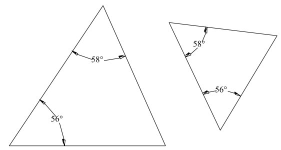
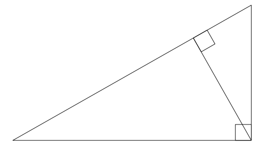
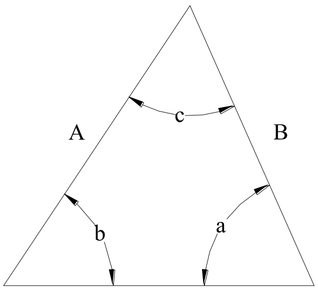
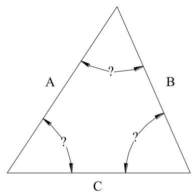
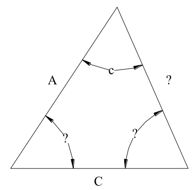
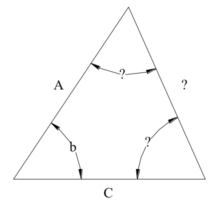
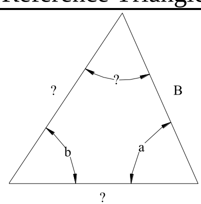
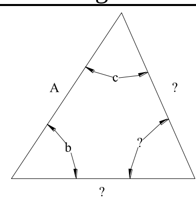
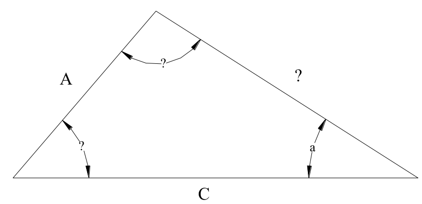
$\sin\theta = \frac{opp}{hyp} = \frac{b}{c}$

$\cos\theta = \frac{adj}{hyp} = \frac{a}{c}$

Procedure Summary:

The triangle is a **Right Triangle**	The triangle is **NOT** a **Right Triangle**
If **two sides** are known, find the third side by using the **Pythagorean Theorem**.	If three parts of **three sides and one angle** are known use **Law of Cosines** to find the fourth part.
If one complementary angle is known find the other using **complements sum to 90°**.	$A^2 + B^2 - 2AB\cos(\theta) = C^2$
If **two sides** are known **find an angle** with:	$\cos(\theta) = \frac{A^2 + B^2 - C^2}{2AB}$
$\theta = \sin^{-1}\frac{opp}{hyp}$, $\theta = \cos^{-1}\frac{adj}{hyp}$, $\theta = \tan^{-1}\frac{opp}{adj}$.	
If **one angle and one side** are known **find another side** using:	If three parts of **two sides and two angles** are known use **Law of Sines** to find the fourth part.
$\sin(\theta) = \frac{opp}{hyp}$, $\cos(\theta) = \frac{adj}{hyp}$, $\tan(\theta) = \frac{opp}{adj}$	$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$

Examples

 Imbedded Triangles with P‖Q Use Similar Triangles	 Reflected Triangles with P‖Q Use Similar Triangles	 Two Angles Match. Use 180° in triangles Use Similar Triangles	 Mitered Right Triangles Use Similar Triangles Use Pythagorean Thm.
 Reference Triangle	 3 Sides, Angles Missing Use Law of Cosines to find angles	 2 Side-Angle Combos Use Law of Sines to find a Many choices after that	 Side-Angle-Side Use Law of Cosines to find third side
 Angle- Angle-Side Use 180° in triangle Use Law of Sines to find sides	 Angle-Side-Angle Use 180° in triangle Use Law of Sines to find sides	 Be careful when using Law of Sines with Obtuse Triangles!	