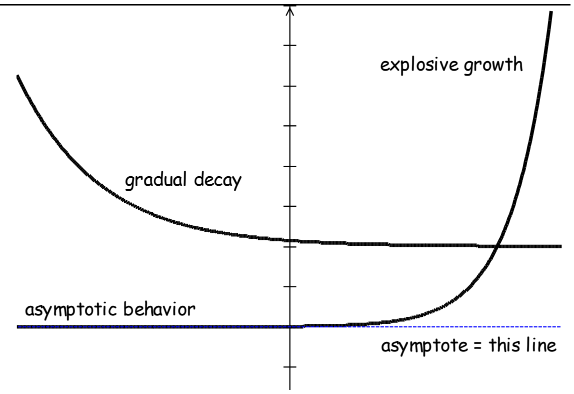
Properties of Exponential Functions

Exponential functions tend to exhibit explosive growth and/or gradual decay.

They tend to exhibit asymptotic behavior: That is f(x) becomes nearly linear. The asymptotes are the lines.

Our basic exponential function has the form:

$$y = Ab^x + C, \quad b > 0$$



Exponential functions arise when increase/decrease is due to a percentage of the population reproducing/dying. Linear functions are associated with a constant increase/decrease (m).
For example:

(a) Suppose you start a fish farm with 500 fish and each week you harvest 10 fish. Then the fish population would be given by $F(t) = 500 - 10t$

(b) Suppose you start a fish farm with 500 fish and each week 5% reproduce. This results in exponential growth and the fish population would be given by $F(t) = 500\,(1 + 5\%)^t$

(c) Suppose you start a fish farm with 500 fish and each week 5% reproduce but you harvest 10% of the fish. This would result in overall exponential decline and the fish population would be given by $F(t) = 500\,(1 + 5\% - 10\%)^t$

Consider the above models of the form $P(t) = P_0\,a^t$. Then, P_0 = initial 'population'. The base 'a' should be written as $a = 1 \pm r$. 'r' should be thought of as a percent. Then 'r' represents the rate of growth/decay. That is, r = percentage of the population that reproduces ($r > 0$) or dies ($r < 0$).

(a) $P = 2000\,(1.07)^t = 2000\,(1 + 7\%)^t$ indicates that the population began at $t = 0$ with 2000 and experiences a 7% growth rate each cycle.

(b) $P = 800\,(0.95)^t = 800\,(1 - 5\%)^t$ indicates that the population began at $t = 0$ with 800 and experiences a 5% reduction rate each cycle.

Exponential Functions of Note

$y = Ab^x + C, \; b > 0$	$y = C$ is the asymptote, $A + C$ = y-intercept.
$P(t) = P_0\,a^t$	P_0 = initial population. $0 < a < 1$ decay, $a > 1$ growth
$P(t) = P_0\,(1 \pm r)^t$	No compounding during a single cycle. $\pm r$ = growth/decline rate
$P(t) = P_0\,(1 + APR/n)^{nt}$	Compounding 'n' times per year
$P(t) = P_0\,e^{rt}$	Base 'e' represents Continuous growth/decay
$P(t) = \dfrac{A}{1 + C\,b^x}$	S-shaped Logistic Function. When mortality increases w/ pop growth.
$N(t) = N_0\,e^{-rt}$	Radioactive Decay. $(r)(HL) = \ln 2$