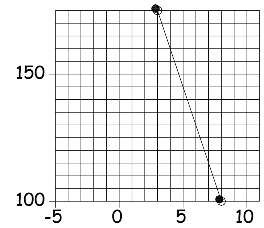
Mth 111 **Pre Exam 3** *Franz Helfenstein* **Name**

1) Use the indicated points to find the equation of this line in slope-intercept form.



2) Solve for x: $3 - \dfrac{x-1}{2} = \dfrac{x}{4}$

3) Solve for x: $\dfrac{3-x}{2} + \dfrac{2}{3} = 1 - \dfrac{3\cdot(2x-5)}{2}$

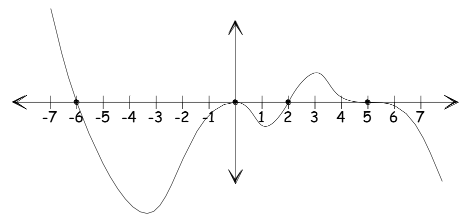
4) Solve for x: $\dfrac{3x^2}{2} - \dfrac{5}{4} = \dfrac{x+3}{4}$

5) Solve for b $\dfrac{a-b+c}{4b} = 10a$

6) Use the polynomial, $P(x) = 2x(x-3)^2(x+2)^3$ to answer the following.

(a) What is the degree of P(x)? (b) What are the roots for P(x)?

7) Give the general form of this polynomial



8) For $f(x) = 1 - 3x^2$, compute and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

9) For $f(t) = t^2$, simplify $4\,f(2x-1) + 10$

10) $P(t) = P_0e^{-kt}$ models radioactive decay. Suppose you start with 22 grams of radioactive Iodine with a half-life of 5 days.

(a) What is the value for P_0? (b) Determine the value for k. Write k to 4 decimal accuracy.

11) $P(t) = 435\,e^{-0.26t}$ models the decline of a coyote population with t in yrs.

(a) What will the coyote population be in 23 yrs? (b) How many years until the coyote population is reduced to 180?

12) Graph $y = -0.1x^3 - 3x^2 - 27.5x - 65$ and adjust the viewing window to see all roots, y-intercepts and local extrema. Then give the type of critical point and its value.

Type of Critical Point				
Value				

13) $f(x) = e^{(2x)} + x$ Simplify the following: (a) $f(3t) =$ (b) $f(a+b) =$

14) Solve for x: (a) $4e^{ax} - 5 = b$ (b) $e^{2x}\,e^{3x+b} = c$ (c) $e^x\,e^{2x+1} = a$

15) Solve for x: (a) $10\ln(ax+b) + 90 = 100$ (b) $\ln(x+4) + \ln(x-2) = \ln 7$ (c) $\ln(ax+b) = -24$

16) A bacteria population at 12:00 was 116,600. 5 hours later the population had increased to 143,053.

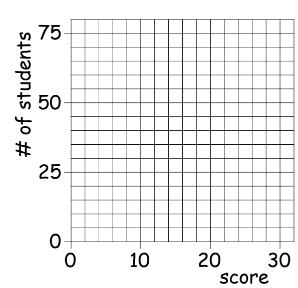
a) Using $P(t) = P_0e^{kt}$ with 12:00 corresponding to t = 0, determine P_0.

b) Determine k accurate to 4 significant digits.

c) Using your model what will be the population in 15 hours?

d) In how many hours will the population reach 250,000?

17) The number of students (P) who got x problems correct out of 30 total is given by $P(x) = 50\,e^{-0.01(x-15)^2}$.



(a) Draw the graph of P(x).

(b) How many students got a perfect score?

(b) How many students got a 0?

18) Mathematics often requires solving complex equations where algebraic methods are insufficient. e.g. This equation would be rather difficult to solve algebraically: $xe^x = 2x^3(x+1)$

<u>**Outline a procedure**</u> for solving such equations, then solve the equation!

19) Simplify to an equivalent expression.

(a) Combine factors and convert to all positive exponents.

$$\frac{(a^2x^{-3})^3\,(ax^4)^2}{(a^{-4}x^0)^2}$$

(b) Simplify to a single term

$$2x + \ln\frac{e^x}{x} + \ln x$$

(c) Simplify to a single number

$$\log_2 160 - \log_2 10$$

20) (a) $\log_5 625 =$ (b) $\log_2 256 =$ (c) $\log_3 (1/27) =$ (d) $\log_{1.5} 2.25 =$

21) Rewrite in an alternative form and simplify where obvious. (a) $\ln xe^x =$ (b) $\log_2 A + 5\log_2 B =$

(c) $\ln 1 - 2\ln (1/e) =$ (d) $\ln 1 + 2 \ln 1 + 3 \ln 1 + \ldots 100 \ln 1 =$ (e) $\log 3x + \log 4x - \log 12x$

22) Pollution is dissipating naturally.

t	0	1	2	3	4	5
ppb	5000	4750	4500	4275	4000	3800

(a) Run exponential regression on this data to determine the exponential function which closely matches the data.

(b) Convert it to the form $y = Ae^{kt}$

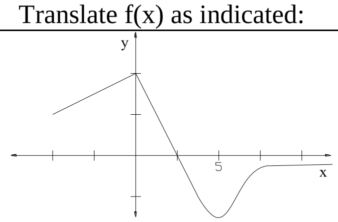
(c) What will the pollution level be when t = 10?

(d) When will the pollution level reach 10 ppb?

23) $f(x) = 3x^2 - 7x$, $g(t) = \sqrt{t + 1}$, $h(w) = w^3$ (a) Compute $f(g(x))$ (b) Compute $2h(g(x)) - g(f(-5))$

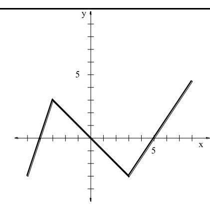
24) Translate f(x) as indicated:

a)



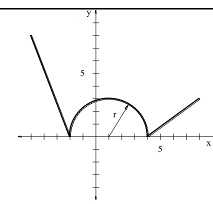
$y = f(x + 3) - 2$

b)



$y = 2\,f(x) - 4$

c)



$y = (½)\,f(2x)$

25) Give the Domain for (a) $y = \dfrac{2 + 3x}{2x - 4} + 1$ (b) $y = \sqrt{x^2 + 1} - 1$ (c) $y = \dfrac{1 + x}{1 - x^2}$ (d) $y = \dfrac{x + 1}{x\sqrt{x - 1}}$

26) If they exist, find the inverses of the following functions: (a) $y = \ln \sqrt{x - 1}$ (b) $y = \dfrac{x + 1}{x - 1}$

(c) $f(x) = 12$ (d) $f(x) = (-¾)x + 24$ (e) $f(x) = \dfrac{12x - 7}{5}$ (f) $y = 1 - \ln (x - 1)$

27) A 4" x 8" plate loses weight by drilling 6 holes in it. Write the plate's weight as a function of the holes' diameter.

28) The half-life of carbon-14 is ≈ 5700 yrs. If a gram of an old relic gives off 12% as much radiation as 1 gram of a similar contemporary piece of the same material how old is the radiation?

29) The half-life of Pu-239, the most common isotope of Plutonium is about 24,000 years. If a site contaminated with Pu-239 must lose 99% of its current radiation level to be considered safe, how long until that site is considered "safe".

30) Assume that with a $0.25/gal gas tax the typical driver will buy 20 gal/wk. At $0.35/gal tax the typical driver will buy 19 gal/wk, at $0.60/gal tax the typical driver will buy 15 gal/wk and at $1.00/gal tax the typical driver will buy 10 gal/wk. (a) Assuming an exponential model, what is the function for gal/wk in terms of tax? (b) At what tax rate will the typical driver purchase only 5 gal/wk? (c) What would the typical purchase be at $5.00/gal tax?

Pre Exam 3 Answers

1) $m = -15, b = 220; y = -15x + 220$

2) $x = 14/3$

3) $x = 38/15$

4) $6x^2 - x - x = 0$; x = (b) $\frac{3x^2}{2} - \frac{5}{4} = \frac{11x + 5}{4}$; $6x^2 - 11x - 10 = 0 = (2x - 5)(3x + 2)$; $x = 5/2, 2/3$

5) $b = \frac{a + c}{40a + 1}$

6) (a) 6^{th} degree (b) $x = -2, 0, 3$

7) $y = -ax^2(x + 6)(x - 2)(x - 5)^3$

8) $m = -6x - 3h$

9) $4f(2x - 1) + 10 = 4(2x - 1)^2 + 10 = 16x^2 - 16x + 14$

10) (a) $P_0 = 22$ (b) $k = \ln(2)/5 \approx 0.1386$

11) (a) $P(23) \approx 1$ coyote (b) 3.39 yrs

12)

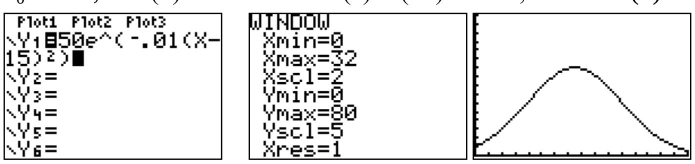
y-int	root (1)	local min	local max
$y = -65$	$x \approx -3.62$	$y \approx 5.19$	$y \approx 14.81$

13) (a) $f(3t) = e^{6t} + 3t$ (b) $f(a + b) = e^{2a + 2b} + a + b$

14) (a) $x = \frac{\ln[(b + 5)/4]}{a}$ (b) $x = \frac{\ln(c) - b}{5}$ (c) $x = \frac{\ln(a) - 1}{3}$

15) (a) $x = \frac{e - b}{a}$ (b) $x^2 + 2x - 15 = 0$; $x = 3$ only (c) no solution, $\ln(ax + b) \geq 0$

16) (a) $P_0 = 116{,}600$ (b) $k \approx 0.04089$ (c) $P(15) \approx 215{,}325$ (d) 18.65 hrs

17) (a) 

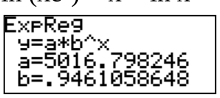
(b) $P(30) = 5$ (c) $P(0) = 5$

18) (a) Enter LHS in Y_1 (b) Enter RHS in Y_2 (c) Graph (d) Find ALL Intersections (2^{nd} CALC) (e) Check that x-values at intersections solve original equation.

19) (a) a^{16}/x (b) $3x$ (c) 4

20) (a) 4 (b) 8 (c) -3 (d) 2

21) (a) $\ln(xe^x) = x + \ln x$ (b) $\log A + 5 \log B = \log(AB^5)$ (c) 2 (d) 0 (e) $\log x$

22) (a) 

(b) A = a, $b = \ln b \approx -0.05540$ (c) $y(10) = 2883$ (d) $t = 112$

23) (a) $f(g(x)) = 3(x + 1) - 7\sqrt{x + 1}$ (b) $2(x + 1)^{3/2} - \sqrt{111}$ (c) $g(f(-3)) = 7$

24) (a) left 3, down 2 (b) y-stretch ×2, down 4 (c) y-compression ×1/2, x-compression ×1/2

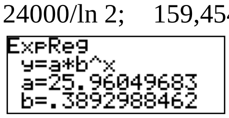
25) (a) $y = \frac{2 + 3x}{2x - 4} + \ln x$; $x > 0, x \neq 2$ (b) x = all reals (c) $x \neq \pm 1$ (d) $x > 1$

26) (a) $y^{-1} = 1 + e^{2x}$ (b) $y^{-1} = = \frac{x + 1}{x - 1}$ (c) y^{-1} does not exist (d) $y^{-1} = (-4/3)x + 8$ (e) $y^{-1} = \frac{5x + 7}{12}$
(f) $y^{-1} = 1 + e^{1 - x}$

27) $W(d) = k[32 - 6\pi(d/2)^2]$, $0 < d < 2$

28) $0.12 = e^{-kt}$ $k = 5700/\ln 2$; 17,436 yrs old

29) $k = 24000/\ln 2$; 159,454 yrs

30) (a) 

(b) $1.75 tax (c) y($5) = 0.23 gal