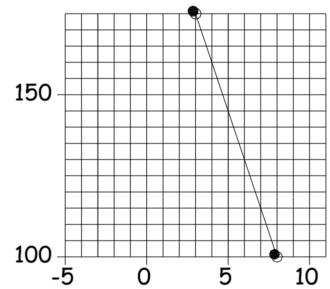
Mth 111 Warmup for Exam 2 *Franz Helfenstein* **Name**



1) Use the indicated points to find the equation of this line in slope-intercept form.

2) Find the equation of the line that passes through the midpoint of the line segment shown here and is perpendicular to this line segment.

3) Solve for x and check your answer: $3 - \frac{x-1}{2} = \frac{x}{4}$

4) Solve for x and check your answer: $\frac{2x-4}{3} - 5 = 12 - \frac{7x-3}{2}$

5) Solve for y: $10 - \frac{2x-5y}{3} = 12 - 5\frac{3y-8x}{2}$

6) Solve for x: $\frac{3x^2}{2} - \frac{5}{4} = \frac{5-7x}{4}$

7) For $f(x) = 1 - 3x^2$, compute and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$

8) For $f(t) = t^2$, simplify $4\,[\,f(x-1)\,] + 10$

9) $f(x) = xe^{(2x)}$ Simplify the following: (a) $f(3t) =$ (b) $f(a+b) =$

10) Simplify to an equivalent expression.

(a) Combine factors and convert to all positive exponents.

$$\frac{(a^2x^{-3})^3\,(ax^4)^2}{(a^{-4}x^0)^2} =$$

(b) Simplify to $a^n a^m$

$a^{mx+b} =$

(c) Simplify to x^n

$f(x) = x^4;\ f^3(x^2) =$

11) A bacteria population at 12:00 was 100. 5 hours later the population had increased to 140.

a) Using $P(t) = P_0e^{kt}$ with 12:00 corresponding to t = 0, determine P_0.

b) Determine k accurate to 4 significant digits.

c) Using your model what will be the population in 15 hours?

d) In how many hours will the population reach 250,000?

12) Mathematics often requires solving complex equations where algebraic methods are insufficient. e.g. This equation would be rather difficult to solve algebraically: $xe^x = 2x^3 (x + 1)$

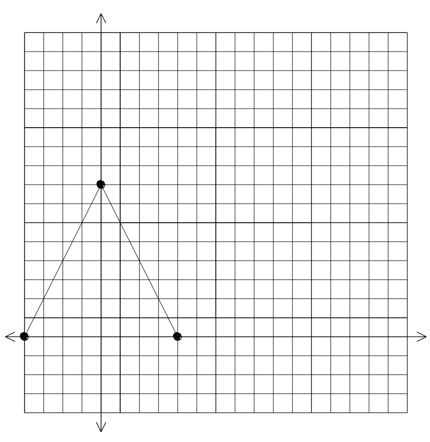
<u>Outline a procedure</u> for solving such equations, then give <u>all solutions</u> to the equation (there are three)!

13) $f(x) = 3x^2 - 12$; $g(t) = \sqrt{t + 1}$

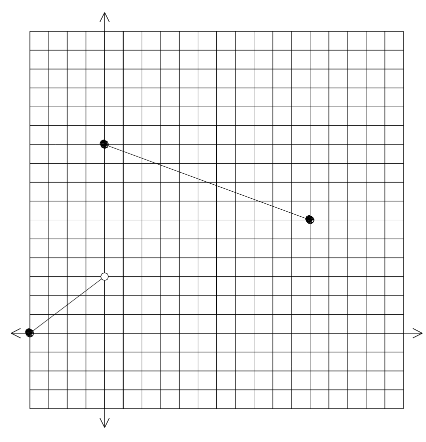
(a) Compute f(g(x)) (b) Compute $f^{\,2}(x)$ (c) Compute $[g(3)/f(2)]^2$

14) Find the inverse of the following function: f(x) = -3x/4 + 24

15) f(x) is shown. (a) Describe 2f(x – 8) – 4 then sketch the result. (b) Describe 8 – f(x) then sketch the result. (c) Describe $f\left(\frac{x-4}{3}\right)$ then sketch the result.



16) g(x) is shown. Find $g^{-1}(x)$ in 2 ways. (a) Draw the inverse and compute the algebraic form. (b) compute the algebraic form for g(x) and compute the inverse from that.

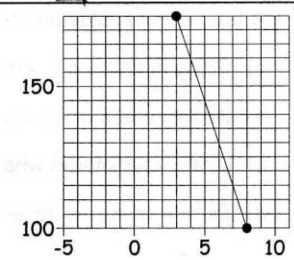


17) Find the average rate of change from x=-2 to x = 6 for y = (¼)(x + 4)(x)(x – 8). What does this number represent? Hint: Use the Table feature of you TI.

18) Find the parabola that passes through (–6, 0) and has a vertex at (8, 6)

Mth 111 Warmup for Exam 2 *Franz Helfenstein* **Name** KEY

1) Use the indicated points to find the equation of this line in slope-intercept form.



$(3,175)\ (8,100)$

$\boxed{y = -15x + 220}$

2) Find the equation of the line that passes through the midpoint of the line segment shown here and is perpendicular to this line segment.

$mp = \left(\frac{3+8}{2}, \frac{175+100}{2}\right)$ $m = \frac{1}{15}$ $\boxed{y = \frac{1}{15}x + \frac{2057}{15}}$

$(11/2,\ 275/2)$

3) Solve for x: $3 + \frac{-x+1}{2} = \frac{x}{4}$

$4 \cdot 3 + \cancel{4}^{2}\ \frac{-x+1}{\cancel{2}} = \frac{x}{\cancel{4}}\ \frac{\cancel{4}}{1}$ $12 + 2 - 2x = x$ $\boxed{x = \frac{14}{3}}$ ✓

$14 = 3x$

$12 + 2(1-x) = x$

4) Solve for x and check your answer: $\frac{2x-4}{3} - 5 = 12 + \frac{-7x+3}{2}$

$\frac{{}^{2}\cancel{6}}{1}\ \frac{2x-4}{\cancel{3}} + 6(-5) = 6 \cdot 12 + \frac{{}^{3}\cancel{6}}{1}\ \frac{3-7x}{\cancel{2}}$ $25x = 119$ $\boxed{x = \frac{119}{25}}$ ✓

$4x - 8 + (-30) = 72 + 9 - 21x$

5) Solve for y: $10 + \frac{-2x+5y}{3} = 12 + (-5)\frac{3y-8x}{2}$ $55y = 124x + 12$

$6 \cdot 10 + \frac{{}^{2}\cancel{6}}{1}\ \frac{5y-2x}{\cancel{3}} = 6 \cdot 12 + \frac{{}^{3}\cancel{6}}{1}\frac{(-5)}{1}\ \frac{3y-8x}{\cancel{2}}$ $\boxed{y = \frac{124x+12}{55}}$

$60 + 10y - 4x = 72 + (-45y) + 120x$

6) Solve for x: $\frac{3x^2}{2} - \frac{5}{4} = \frac{5-7x}{4}$

$\frac{{}^{2}\cancel{4}}{1}\ \frac{3x^2}{\cancel{2}} - \frac{\cancel{4}}{1}\ \frac{5}{\cancel{4}} = \frac{\cancel{4}}{1}\ \frac{5-7x}{\cancel{4}}$ $6x^2 + 7x - 10 = 0$ $\boxed{x = -2,\ 5/6}$

$(6x-5)(x+2) = 0$

$6x^2 - 5 = 5 - 7x$

7) For $f(x) = 1 - 3x^2$, compute and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$

$\frac{[1-3(x+h)^2] - [1-3x^2]}{h} = \frac{\cancel{1} + (-3)[x^2 + 2xh + h^2] \cancel{-1} + 3x^2}{h} = \frac{(-6x-3h)h}{h} = \boxed{-6x - 3h}$

8) For $f(t) = t^2$, simplify $4[f(x-1)] + 10$

$4[(t-1)^2] + 10 = 4t^2 - 8t + 4 + 10 = \boxed{4t^2 - 8t + 14}$

9) $f(x) = xe^{(2x)}$ Simplify the following: (a) $f(3t) =$ $\boxed{3te^{6t}}$ (b) $f(a+b) = \boxed{(a+b)e^{2a+2b}}$ or $\boxed{(a+b)e^{2a} \cdot e^{2b}}$

10) Simplify to an equivalent expression.

(a) Combine factors and convert to all positive exponents.

$\frac{(a^2x^{-3})^3(ax^4)^2}{(a^{-4}x^0)^2}$ $\boxed{a^{16}/x}$

(b) Simplify to $a^n b^m$

$(a^3b^4)^2(a^3b^2)^4 =$ $\boxed{a^{18}b^{16}}$

(c) Simplify to x^n

$f(x) = x^4;\ f^3(x^2) =$ $[(x^2)^4]^3 = \boxed{x^{24}}$

11) A bacteria population at 12:00 was 100. 5 hours later the population had increased to 140.

a) Using $P(t) = P_0e^{kt}$ with 12:00 corresponding to t = 0, determine P_0. $P_0 = 100$

b) Determine k accurate to 4 significant digits. $140 = 100e^{k \cdot 5}$ $k \sim 0.06729$

c) Using your model what will be the population in 15 hours? ~ 274,

d) In how many hours will the population reach 250,000? 116.3 hrs

12) Mathematics often requires solving complex equations where algebraic methods are insufficient. e.g. This equation would be rather difficult to solve algebraically: $xe^x = 2x^3(x + 1)$

<u>Outline a procedure</u> for solving such equations, then give <u>all solutions</u> to the equation!

Left Hand Side → y_1 Find intersections $x \cong 0, 0.7834, 6.413$

Right Hand Side → y_2 x-values = solutions

Graph

13) $f(x) = 3x^2 - 12$, $g(t) = \sqrt{t + 1}$

(a) Compute f(g(x)) (b) Compute $f^2(x)$ (c) Compute $[g(3)/f(2)]^2$

$3(\sqrt{t+1})^2 - 12 = \boxed{3t-9}$ $\boxed{(3x^2-12)^2 = 9x^4 - 72x^2 + 144}$ $\dfrac{\sqrt{4}}{3 \cdot 2^2 - 12} = \boxed{\phi}$

14) Find the inverse of the following function: $f(x) = -3x/4 + 24$

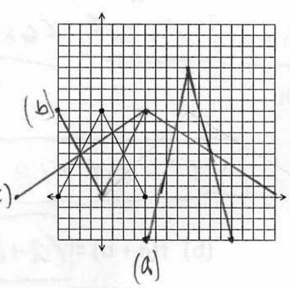
$f^{-1}(x) = \dfrac{4x - 96}{-3} = -\dfrac{4x + 96}{3}$

15) f(x) is shown. (a) Describe 2f(x − 8) − 4 then sketch the result. (b) Describe 8 − f(x) then sketch the result. (c) Describe $f\left(\frac{x-4}{3}\right)$ then sketch the result.

(a) → 8, ↕×2, ↓4

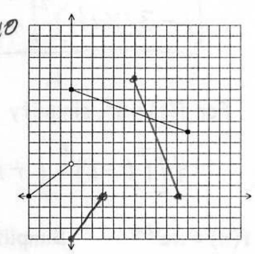
(b) ⤺, ↑8

(c) → 4, ↔ ×3



16) g(x) is shown. Find $g^{-1}(x)$ in 2 ways. (a) Draw the inverse and compute the algebraic form. (b) compute the algebraic form for g(x) and compute the inverse from that.

$g^{-1}(x) = \begin{cases} \frac{4}{3}x - 4, & 0 \le x < 3 \\ \frac{-11x}{4} + 27.5, & 6 \le x \le 10 \end{cases}$



17) Find the average rate of change from x=−2 to x = 6 for $y = (\frac{1}{4})(x + 4)(x)(x - 8)$. What does this number represent? Hint: Use Tables

(−2, 10)
(6, −30)

$m = \dfrac{-30-10}{6 - -2} = \boxed{-5}$ Slope from x = −2 to x = 6 on curve.

18) Find the parabola that passes through (−6, 0) and has a vertex at (8, 6) = (h, k)

$y = a(x-8)^2 + 6$ $a = -3/98$

$0 = a(-6-8)^2 + 6$

$\boxed{y = \frac{-3}{98}(x-8)^2 + 6}$

pg 2