Mth 111 Practice with Function Notation *Franz Helfenstein* Name

Use $f(x) = 2x^2 - 1$, $g(x) = \sqrt{x + 1}$, $m(x) = \dfrac{1 - x}{2}$ to rewrite each of the following. Simplify where possible.

Note: f(new): Always begin with (new) and go from there. For complex expressions, work from inside out.

1) $f(x + h) =$

1a) $f(x) + h =$

2) $\dfrac{f(x + h) - f(x)}{h} =$

3) $m(x - k) =$

3a) $m(x) - k =$

4) $\dfrac{m(x - k) - m(x)}{k} =$

5) $f^2(x) =$

5a) $f(x^2) =$

6a) $f(g(x)) =$

6b) $g(f(x)) =$

7) $4\,f(2x + 1) + 15 =$

8) $g(m(x)) =$

$m(g(x)) =$

9) $g(m(x - 1)) =$

10a) $m^{-1}(x) =$

10b) $m(x^{-1}) =$

10c) $m(x)^{-1} =$

11) $m^{-1}(x + 2) =$

12) $10 - f(x^2) =$

13) $f(1)\, f(2)\, f(3) =$

14) $g^2(x) =$

15a) $f(x + 1) =$

15b) $f(x) + 1 =$

16) $[f(x)]^{-1} =$

17) $g^7(x - 1) =$

18) $10\, f(x) + 4\, m(x) =$

19) $5\, f(3x) + 5 =$

20) $10\, f(x + 2) + 5x =$

21) $x\, f(x) =$

22) $x^5\, f(x^2) =$

23) $f(e^x) =$

24) $\ln (g(x)) =$

25) $f(\ln x) =$

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Use $f(x) = 2x^2 - 1$, $g(x) = \sqrt{x+1}$, $m(x) = \dfrac{1-x}{2}$ to rewrite each of the following. Simplify where possible.

1) $f(x+h) = 2(x+h)^2 - 1 = 2x^2 + 4xh + 2h^2 - 1$

1a) $f(x) + h = [f(x)] + h = [2x^2 - 1] + h = 2x^2 - 1 + h$

2) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{[f(x+h)] - [f(x)]}{h} = \dfrac{[2(x+h)^2 - 1] - [2x^2 - 1]}{h} = \dfrac{h(4x + 2h)}{h} = 4x + 2h$

3) $m(x-k) = \dfrac{1-(x-k)}{2} = \dfrac{1-x+k}{2}$

3a) $m(x) - k = [m(x)] - k = \left[\dfrac{1-x}{2}\right] - k = \dfrac{1-x-2k}{2}$

4) $\dfrac{m(x-k) - m(x)}{k} = \dfrac{[m(x-k)] - [m(x)]}{k} = \dfrac{\left[\dfrac{1-(x-k)}{2}\right] - \left[\dfrac{1-x}{2}\right]}{k} = \dfrac{1-x+k-1+x}{2} \cdot \dfrac{1}{k} = 1/2$

5) $f^2(x) = [f(x)]^2 = [2x^2 - 1]^2 = 4x^4 - 4x^2 + 1$

5a) $f(x^2) = 2(x^2)^2 - 1 = 2x^4 - 1$

6a) $f(g(x)) = f(\sqrt{x+1}) = 2\left(\sqrt{x+1}\right)^2 - 1 = 2x + 1$

6b) $g(f(x)) = g(2x^2 - 1) = \sqrt{(2x^2 - 1) + 1} = x\sqrt{2}$

7) $4f(2x+1) + 15 = 4[f(2x+1)] + 15 = 4[2(2x+1)^2 - 1] + 15 = 32x^2 + 32x + 19$

8) $g(m(x)) = g\left(\dfrac{1-x}{2}\right) = \sqrt{\dfrac{1-x}{2} + 1} = \sqrt{\dfrac{1-x}{2} + \dfrac{2}{2}} = \sqrt{\dfrac{3-x}{2}}$

$m(g(x)) = \dfrac{1 - \sqrt{x+1}}{2}$

9) $g(m(x-1)) = g\left(\dfrac{2-x}{2}\right) = \sqrt{\dfrac{2-x}{2} + 1} = \sqrt{\dfrac{4-x}{2}}$

$m(x-1) = \dfrac{1-(x-1)}{2} = \dfrac{2-x}{2}$

10a) $m^{-1}(x) = 1 - 2x$

10b) $m(x^{-1}) = \dfrac{1-(x^{-1})}{2} = \dfrac{1-(1/x)}{2} = \dfrac{x-1}{2x}$

10c) $m(x)^{-1} = \varnothing$

$x = \dfrac{1-y}{2} \rightarrow y = 1 - 2x$

11) $m^{-1}(x+2) = 1 - 2(x+2) = -2x - 3$

12) $10 - f(x^2) = 10 - [f(x^2)] = 10 - [2(x^2)^2 - 1] = 10 - 2x^4 + 1 = 11 - 2x^4$

13) $f(1)\,f(2)\,f(3) = [2(1)^2 - 1]\,[2(2)^2 - 1]\,[2(3)^2 - 1] = 119$

14) $g^2(x) = [g(x)]^2 = \left[\sqrt{x+1}\right]^2 = x + 1$

15a) $f(x+1) = 2(x+1)^2 - 1 = 2x^2 + 4x + 1$

15b) $f(x) + 1 = [f(x)] + 1 = [2x^2 - 1] + 1 = 2x^2$

16) $[f(x)]^{-1} = \dfrac{1}{f(x)} = \dfrac{1}{2x^2 - 1}$

17) $g^7(x-1) = [g(x-1)]^7 = \left[\sqrt{(x-1)+1}\right]^7 = \sqrt{x}^{\,7} = x^{7/2}$

18) $10\,f(x) + 4\,m(x) = 10[2x^2 - 1] + 4\left[\dfrac{1-x}{2}\right] = 20x^2 - 2x - 8$

19) $5\,f(3x) + 5 = 5[2(3x)^2 - 1] + 5 = 90x^2$

20) $10\,f(x+2) + 5x = 10[2(x+2)^2 - 1] + 5x = 20x^2 + 85x + 70$

21) $x\,f(x) = x[2(x)^2 - 1] = 2(x)^3 - x$

22) $x^5\,f(x^2) = x^5[2(x^2)^2 - 1] = 2x^9 - x^5$

23) $f(e^x) = 2(e^x)^2 - 1 = 2e^{2x} - 1$

24) $\ln(g(x)) = \ln(\sqrt{x+1}\,) = (\tfrac{1}{2})\ln(x+1)$

25) $f(\ln x) = 2(\ln x)^2 - 1 = 2\ln^2(x) - 1$

$f(\#) = 2(\#)^2 - 1$ $\quad f(\Diamond) = 2(\Diamond)^2 - 1$ $\quad f(\Delta) = 2(\Delta)^2 - 1$ $\quad f(\text{Ж} + \text{Ш}) = 2(\text{Ж} + \text{Ш})^2 - 1$